General exam data			
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If there are multiple versions: VERSION: [ ]

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Please write your name and student number on each sheet. Good luck!

Resit Exam: Life Insurance (35V6A3-B-6)

Date: 09-07-2024.

Duration: 3 hours.

This exam has 11 pages (not including cover pages).

Give complete answers with sufficient explanations

Question 1 (30=4+8+6+6+6pts): In this exercise, we consider a *modification* of the death benefit for an insured (x,t). Here, we consider a couple (x,t) and (y,t) with respective independent remaining lifetimes $T_{x,t}$ and $T'_{y,t}$. Assume: $x = y = 75$, $t = 2024$. In your expressions, you explicitly assume a flat term structure with discount factor $\nu = 0.95$. The classical death benefit product with ten possible payment dates is modified/defined in the following way:

- There is a waiting time of five years: there will be no payment in case the insured (x,t) dies in the first five years. The total *number* of possible payment dates still equals $K = 10$: so, the final possible payment is in 2039.
- When (x,t) dies within the pre-specified time frame, the payoff $C = 15,000$ will only be paid if the partner is still alive at the payment date.

(a) Express the yearly compounded interest rate and of the continuously compounded interest rate in terms of the discount factor.

Solution:

(i) discrete compounding: $v = \frac{1}{1+r}$. (2)

(ii) continuous compounding: $v = \mathrm{e}^{-R_t(t+1)}$. (2)

(b.i) Explain why insurers might want to introduce a waiting period.

Solution: A waiting period avoids costly health checks. (1)

This also prevents people who are at acute risk of suicide or have a critical illness from taking out life insurance. (1)

(b.ii) Give the pay-off stream $\{Y(k)\}_{k\geq 0}$ of this contract.

Solution:

$$Y(k) = \underbrace{C}_{(0.5)} \underbrace{1_{\{T_{x,t}=k-1\}}}_{(1)} \underbrace{1_{\{T_{y,t}\geq k\}}}_{(1)} \underbrace{1_{\{6\leq k\leq 15\}}}_{(0.5)}$$

(b.iii) Determine an expression for the net lump sum premium.

Solution:

$$\pi(Y) = \underbrace{C}_{(0.5)} \underbrace{\sum_{k=6}^{15}}_{(0.5)} \underbrace{{}_{k-1}p_{x,t}}_{(0.5)} \cdot \underbrace{q_{x+k-1,t+k-1}}_{(0.5)} \cdot \underbrace{{}_{k}p_{y,t}}_{(0.5)} \cdot \underbrace{v^k}_{(0.5)}$$

(c) Consider the case of net periodic premiums. Determine π with π the constant premium to be paid by the insured (x, t) at the beginning of each year but only if a future benefit payment is possible to the partner (y, t).

Solution: Payment stream:

$$\pi(k) = \underbrace{\pi}_{(0.5)} \underbrace{1_{\{T_{x,t} \geq k\}}}_{(1)} \underbrace{1_{\{T_{y,t} \geq k\}}}_{(1)} \underbrace{1_{\{0 \leq k \leq 14\}}}_{(0.5)}$$

Actuarially fair premium:

$$\pi = \frac{\pi(Y) \quad (1)}{\underbrace{\sum_{k=0}^{14}}_{(0.5)} \underbrace{{}_{k}p_{x,t}}_{(0.5)} \underbrace{{}_{k}p_{y,t}}_{(0.5)} \underbrace{v^k}_{(0.5)}}$$

An insurer, with no initial reserves, has N identical, independent contracts as discussed above. Each insured pays the net lump sum premium, say $\pi(Y)$ (as determined in (a.iii)), plus a surcharge of $100\delta\%$ (where $\delta > 0$). The resulting time t reserves are invested in short bonds with a maturity of 1 year. Just as above, there is no interest rate risk and the term structure is determined by the discount factor $\nu = 0.95$. The financial regulator requires that the asset liability ratio at time $t+1$ is at least 100% with probability larger than or equal to $100(1-\alpha)\%$ for some given value $\alpha \in (0, 1)$.

(d.i) Give an expression for the value of the reserves A_0, A_1 at time t and $t+1$, respectively.

Solution: Reserves:

$$A_0 = N(1 + \delta)\pi(Y) \qquad (1)$$

$$A_1 = \frac{A_0}{v} \qquad (1)$$

(d.ii) Determine the expected pay-off stream given the time $t+1$ information $\mathcal{F}_1$ about the remaining lifetimes of (x,t) and (y,t) at time $t+1$: that is, determine an expression for $\{E(Y_i(k)|\mathcal{F}_1)\}_{k\geq 1}$ for each couple $i=1,\ldots,N$.

Solution:

$$E(Y_i(k)|\mathcal{F}) = \underbrace{C}_{\textcircled{0.5}} \underbrace{1_{\{T_{y,t}\geq 1, T_{x,t}\geq 1, 6\leq k\leq 15\}}}_{\textcircled{0.5}} \underbrace{{}_{k-2}p_{x+1,t+1}q_{x+k-1,t+k-1}}_{\textcircled{0.5}} \cdot \underbrace{{}_{k-1}p_{y+1,t+1}}_{\textcircled{0.5}}$$

(d.iii) Give an expression for the expected discounted liabilities L_1 at time $t+1$.

Solution:

$$L_1 = \underbrace{\sum_{i=1}^{\infty}}_{\textcircled{0.5}} \underbrace{1_{\{T^i_{x,t}\geq 1, T^i_{y,t}\geq 1\}}}_{\textcircled{0.5}} \frac{\pi(Y)}{\underbrace{p_{x,t}p_{y,t}v}_{\textcircled{1}}}$$

(e.i) Suppose $N=1$ and assume there is no macro longevity risk. Show that the regulator's requirement is met in case $p_{x,t}\cdot p_{y,t}\leq\alpha$.

Solution:

$$\begin{aligned}\mathbb{P}(L_1\leq A_1) &= \mathbb{P}\Big(1_{\{T^i_{x,t}\geq 1, T^i_{y,t}\geq 1\}}\frac{\pi(Y)}{p_{x,t}p_{y,t}v}\leq\frac{(1+\delta)\pi(Y)}{v}\Big)\\ &= P\Big(1_{\{T^i_{x,t}\geq 1, T^i_{y,t}\geq 1\}}\leq(1+\delta)p_{x,t}p_{y,t}\Big) \qquad \textcircled{1}\\ &\geq P\Big(1_{\{T^i_{x,t}\geq 1, T^i_{y,t}\geq 1\}}\leq 0\Big)\\ &= 1-p_{x,t}p_{y,t}\geq 1-\alpha \qquad \textcircled{1}\end{aligned}$$

(e.ii) Is the requirement $p_{x,t}\cdot p_{y,t}\leq\alpha$ necessary for all values of δ?

Solution: No! (0.5)

For instance, in case

$$\alpha < p_{x,t}p_{y,t} < \frac{1}{1+\delta} \qquad \textcircled{1}$$

we have $\mathbb{P}(L_1\leq A_1) =< 1-\alpha$. (0.5) (e.iii) How would you verify the regulator's requirement for large values of N?

Solution: Perform a Monte-Carlo simulation and simulate L_1 and A_1 say 100,000 times to estimate the probability $\mathbb{P}(L_1 \leq A_1)$. (2)

Question 2 (20=6+14pts):

(a.i) Explain the idea of the Nelson-Siegel-Svensson model for the term structure of interest rates and compare it to the Vasicek model.

Solution: The Nelson-Siegel-Svensson model builds on the Nelson Siegel model and models the term structure of interests with four factors. (1)

These factors determine the slope, the shape, the location of the humps of the term structure. (1)

By contrast, Vasicek is a mean-reverting stochastic one-factor model for the term structure of interest rates. (1)

In Vasicek, the term structure materializes in equilibrium, while in Nelson-Siegel-Svensson, it is exogenously given. (1)

(a.ii) Describe how the parameters in the Nelson-Siegel-Svensson model are obtained from market data.

Solution: The Nelson-Siegel-Svensson termstructure is parametrized by a set of six parameters Θ. If the corresponding term structure is denoted by $R_t^{\Theta}(t+k)$, and the observable real-world term structure is denoted by $R_t(t+k)$, one can easily estimate the parameter set by solving

$$\widehat{\Theta} = \arg\min_{\Theta} \sum_{i=1}^{N} [R_t^{\Theta}(t+T_i) - R_t(t+T_i)]^2. \quad (2)$$

(b) In the Vasicek model the term structure of interest rates is explained by a single factor $\{X_t\}$ with dynamics:

$$X_{t+1} = \mu + \theta X_t + \sigma \varepsilon_{t+1}, \text{ with } \varepsilon_{t+1} \sim N(0,1) \ i.i.d., \ \theta \in [0,1], \ \sigma > 0.$$

For each $k \in \mathbf{N}$, it can be shown that the current price of a zero coupon bond with maturity date $t+k$ is given by $P_t(t+k) = e^{-A_k - B_k X_t}$, where $A_0 = B_0 = 0$, $B_k = 1 + \theta B_{k-1}$, and $A_k = A_{k-1} + \mu^{\mathcal{Q}} B_{k-1} - \frac{1}{2}\sigma^2 B_{k-1}^2$. Let $R_t(t+k)$ denote the corresponding yield of this zero coupon bond.

(i) Interpret the factor X_t.

Solution: X_t is the short-term spot rate holding for a period of length ΔT, i.e., it is revealed at period t and holds for an investment from t to $t+1$. (2)

(ii) Suppose $\theta < 1$. Show: $B_k \to \frac{1}{1-\theta}$ and $\frac{A_k}{k} \to \mu^{\mathcal{Q}}\frac{1}{1-\theta} - \frac{1}{2}\sigma^2\frac{1}{(1-\theta)^2}$ as $k \to \infty$.

Solution: We have $B_k = \frac{1-\theta^k}{1-\theta}$. Taking the limit for $k \to \infty$ yields the desired results. (1)

Making use of the telescope sum property, we obtain

$$\frac{A_k}{k} = \frac{1}{k}\sum_{i=1}^{k}(A_i - A_{i-1}) = \frac{1}{k}\sum_{i=1}^{k}\left(\mu^{\mathcal{Q}}B_{k-1} - \frac{1}{2}\sigma^2 B_{k-1}^2\right) \quad (1)$$
$$\to \mu^{\mathcal{Q}}\frac{1}{1-\theta} - \frac{1}{2}\sigma^2\frac{1}{(1-\theta)^2} \quad (1)$$

as k tends to infinity.

(iii) Suppose $\theta < 1$. Show that the limit of $R_t(t+k)$ when $k \to \infty$ does not depend on X_t. Also determine this limit.

Solution: Notice that $\lim_{k\to\infty} B_k = \frac{1}{1-\theta}$ and $\lim_{k\to\infty}\frac{B_k}{k} = 0$. (1)

Consequently,

$$\lim_{k\to\infty} R_t(t+k) = \lim_{k\to\infty}(1 - \frac{B_k}{k})(\frac{\mu^{\mathcal{Q}}}{1-\theta} - \frac{1}{2}\frac{\sigma^2}{(1-\theta)^2}) + \frac{B_k}{k}X_t + \frac{1}{2}\sigma^2\frac{1}{1+\theta}\frac{1}{1-\theta}\frac{B_k(B_k-1)}{k}$$
$$= \frac{\mu^{\mathcal{Q}}}{1-\theta} - \frac{1}{2}\frac{\sigma^2}{(1-\theta)^2}. \quad (1)$$

(iv) Suppose $\theta = 1$. Determine, for each $k \in \mathbf{N}$, a closed form expression for B_k and A_k. [Hint: without proof you may use, for $n \in \mathbf{N}$, $1 + 2 + 3 + \cdots + n = \frac{1}{2}n(n+1)$ and $1 + 4 + 9 + \cdots + n^2 = \frac{1}{6}(2n+1)n(n+1)$.]

Solution: Obviously, $B_k = k$. (1)

$$A_k = \sum_{i=1}^{k}\left(\mu^{\mathcal{Q}}B_{k-1} - \frac{1}{2}\sigma^2 B_{k-1}^2\right)$$
$$= \sum_{i=1}^{k}\left(\mu^{\mathcal{Q}}(k-1) - \frac{1}{2}\sigma^2(k-1)^2\right) \quad (1)$$
$$= \frac{1}{2}\mu^{\mathcal{Q}}(k-1)k - \frac{1}{12}\sigma^2(2k-1)(k-1)k \quad (1)$$

(v) Suppose $\theta = 1$. Determine the limit of $R_t(t+k)$ when $k \to \infty$. **Solution:** The term structure is given by

$$R_t(t+k) = \frac{A_k}{k} + \frac{B_k}{k}X_t \quad (1)$$
$$\to \infty \quad (1)$$

in case $\theta = 1$.

(vi) Give several reasons why $\theta = 1$ does not result in a reasonable model for X_t. **Solution:**

- ∗ Exploding term structure. ①
- ∗ short rate is a random walk with drift, hence exploding. ①

***** Start Questions 3 and 4 on a new answer sheet *****

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Question 3 (30=5+5+5+5+6+4 pts):
In the Lee-Carter model, the central death rate evolves according to

$$\ln(m_{x,t}^{(g)}) = \alpha_x^{(g)} + \beta_x^{(g)}\kappa_t^{(g)} + \varepsilon_{x,t}^{(g)}, \tag{1}$$

where κ_t follows a random walk with drift, i.e., $\kappa_t^{(g)} = c^{(g)} + \kappa_{t-1}^{(g)} + \delta_t^{(g)}$. Suppose you have estimated the Lee-Carter model under the Liu-et-al.-normalization, i.e., $\sum_{x\in\mathcal{X}} \beta_x^{(g)} = 1$ and $\sum_{x\in\mathcal{X}} \alpha_x^{(g)} = 0$.

(a) Show analytically that without appropriate normalizations, the estimates are not uniquely determined.

Solution: Assume that $\widehat{\alpha}_x^{(g)}, \widehat{\beta}_x^{(g)}, \widehat{\kappa}_t^{(g)}$ minimize the sum of squared errors

$$\sum_{x,t} \left[\ln(m_{x,t}^{(g)}) - \widehat{\alpha}_x^{(g)} - \widehat{\beta}_x^{(g)}\widehat{\kappa}_t^{(g)}\right]^2, \qquad (1)$$

then also $\widetilde{\alpha}_x^{(g)} = \widehat{\alpha}_x^{(g)} + c_1\widehat{\beta}_x^{(g)}$, $\widetilde{\beta}_x^{(g)} = c_2\widehat{\beta}_x^{(g)}$, $\widetilde{\kappa}_t^{(g)} = \frac{\widehat{\kappa}_t^{(g)}}{c_2} - \frac{c_1}{c_2}$: (2)

$$\widehat{\alpha}_x^{(g)} + \widehat{\beta}_x^{(g)}\widehat{\kappa}_t^{(g)} = \widetilde{\alpha}_x^{(g)} + \widetilde{\beta}_x^{(g)}\widetilde{\kappa}_t^{(g)}. \qquad (1)$$

Consequently, the maximizers are not uniquely determined. (1)

(b) Show how to transform $\alpha_x^{(g)}, \beta_x^{(g)}, \kappa_t^{(g)}$ from the Liu-et-al.-normalization to the standard Lee-Carter normalization.

Solution: Liu-et-al.-normalization $\sum_{x\in\mathcal{X}} \beta_x^{(g)} = 1$ and $\sum_{x\in\mathcal{X}} \alpha_x^{(g)} = 0$.

Lee-Carter normalization $\sum_{x\in\mathcal{X}} \beta_x^{(g)} = 1$ and $\sum_{t\in\mathcal{T}} \kappa_t^{(g)} = 0$.

We define

$$\widehat{\beta}_x^{(g)} = \beta_x^{(g)} \qquad (1)$$
$$\widehat{\kappa}_t^{(g)} = \kappa_t^{(g)} - \frac{1}{T}\sum_{s\in\mathcal{T}} \kappa_s^{(g)} \qquad (1)$$
$$\widehat{\alpha}_x^{(g)} = \alpha_x^{(g)} + \beta_x^{(g)}\frac{1}{T}\sum_{s\in\mathcal{T}} \kappa_s^{(g)} \qquad (1)$$

Then, obviously, $\sum_{x\in\mathcal{X}} \widehat{\beta}_x^{(g)} = 1$ (0.5). Besides, $\sum_{t\in\mathcal{T}} \widehat{\kappa}_t^{(g)} = 0$ (0.5) And it holds that

$$\alpha_x^{(g)} + \beta_x^{(g)}\kappa_t^{(g)} = \widetilde{\alpha}_x^{(g)} + \widetilde{\beta}_x^{(g)}\widetilde{\kappa}_t^{(g)}. \qquad (1)$$

(c) Explain the intuition behind the two different normalizations.

Solution: Lee-Carter normalization: $\sum_{t\in\mathcal{T}} \widehat{\kappa}_t^{(g)} = 0$ means that on average, the dynamic age effect is zero. ①

Thus, in the middle of the life-table (currently around 1992, everything is captured in the age-fixed effects $\alpha_x^{(g)}$. ①

In the Liu-normalization, on average, the age-fixed effect is zero $\sum_{x\in\mathcal{X}} \alpha_x^{(g)} = 0$. ①

for the average age group, everything is captured in the dynamic age effects $\beta_x^{(g)}\kappa_t^{(g)}$. ①

$\sum_{x\in\mathcal{X}} \widehat{\beta}_x^{(g)} = 1$ is an ad-hoc assumption with no deeper economic meaning. ①

(d) Derive the maximum likelihood estimator for $c^{(g)}$ and for $\mathrm{var}(\delta_t^{(g)})$.

Solution: Rewrite the equation for $\kappa_t^{(g)}$:

$$\Delta\kappa_t^{(g)} = c^{(g)} + \delta_t^{(g)}, \qquad ①$$

The MLE for an expectation is its sample mean, i.e.,

$$\widehat{c}^{(g)} = \frac{1}{T-1}\sum_{t=2}^{T} \Delta\kappa_t^{(g)} = \frac{\kappa_T^{(g)} - \kappa_1^{(g)}}{T-1}. \qquad ②$$

Similarly, the MLE for the variance is the sample variance, i.e.,

$$\widehat{\sigma}_\delta^{(g)} = \frac{1}{T-1}\sum_{t=2}^{T} \left[\Delta\kappa_t^{(g)} - \widehat{c}^{(g)}\right]^2. \qquad ②$$

(e) Instead of modeling the raw central death rate, one could directly model the hazard rate of death, e.g.,

$$\ln(\mu_{x,t}^{(g)}) = \alpha_x^{(g)} + \beta_x^{(g)}\kappa_t^{(g)}. \tag{2}$$

(i) State a definition of the hazard rate of death and explain the relation between the hazard rate of death and the raw central death rate.

Solution: The hazard rate of death is defined as

$$\mu_{x+\tau,t+\tau}^{(g)} = -\frac{\partial}{\partial\tau}\ln({}_\tau p_{x,t}^{(g)}). \qquad ①$$

The *raw central death rate*

$$m_{x,t}^{(g)} = \widehat{\mu}_{x,t}^{(g)} = \frac{D_{x,t}^{(g)}}{E_{x,t}^{(g)}} \qquad ①$$

is the maximum likelihood estimator for $\mu_{x+\tau,t+\tau}^{(g)}$. ①

(ii) Explain how one typically models uncertainty in (2), and outline how the model can be estimated.

Solution: Model the number of deaths $D_{x,t}^{(g)}$ as Poisson-distributed random variables, ①

Given the definition of $m_{x,t}^{(g)}$, a reasonable model is

$$D_{x,t}^{(g)} \mid E_{x,t}^{(g)} \sim \mathcal{P}(\mu_{x,t}^{(g)} E_{x,t}^{(g)}). \qquad ①$$

The model can be estimated by the maximum likelihood method. ①

(f) Suppose someone argues that the Lee-Carter model should not be applied in small populations and instead proposes the following two-layer Lee-Li model

$$\ln(m_{x,t}^{(g)}) = \alpha_x^{(g)} + \beta_{x,1}^{(g)}\kappa_{t,1}^{(g)} + \beta_{x,2}^{(g)}\kappa_{t,2}^{(g)} + \sigma_\epsilon^{(g)}\varepsilon_{x,t}^{(g)} \tag{3}$$

with $\kappa_{t,1}^{(g)} = c^{(g)} + \kappa_{t-1}^{(g)} + \sigma_\delta^{(g)}\delta_t^{(g)}$, and $\kappa_{t,2}^{(g)} = \mu^{(g)} + \theta^{(g)}\kappa_{t-1,2}^{(g)} + \sigma_\eta^{(g)}\eta_t^{(g)}$, where $\varepsilon_{x,t}^{(g)}, \delta_t^{(g)}, \eta_t^{(g)}$ are i.i.d. standard normally distributed shocks. Motivate the modelers choice of a two-layer Lee-Li model and explain why such a model could improve the quality of the forecasts compared to a standard Lee-Carter model.

Solution: The first layer (standard Lee-Carter) models mortality of Europe (or the US), while the second layer $\beta_{x,2}^{(g)}\kappa_{t,2}^{(g)}$ models the deviation of the country-specific (or state-specific) mortality from the European (or a US) mortality, ②

This approach enlarges the data set and thus improves the quality of the estimates ①. Moreover, the second layer can easily be estimated with maximum likelihood techniques. ①

Question 4 (20=7+3+10 pts):
Suppose an insurance company has a developed and sold a new term life insurance product that pays off a lump sum payment P in the event of death of the insured person. This payment is doubled if the insured has died from COVID-19. To get protection, the insured person has to make constant premium payments until death. To model the survival probabilities of the insureds, the life insurer uses the AG2022 model.

(a) Explain carefully how the third layer of the AG2022-model accounts for the excess deaths caused by COVID-19. Elaborate on the use of weekly data to form the COVID-19 risk factor in this model.

Solution: Third layer: $\mathcal{B}_x\mathcal{X}_t$ (1) models the *dynamic* age effect of COVID-19. Compared to a Lee-Li model, the hazard rate of death is increased by $\mathcal{B}_x\mathcal{X}_t > 0$ (1)

Time series $\mathcal{X}_t$ tends to decrease over time and represents the transitory nature of COVID-19 (1). $\mathcal{B}_x$ is age-specific and models the impact of COVID on each age group. (1)

Efficient estimation of the dynamic age-effect requires granular data. (1) The model makes use of weekly date, applies maximum likelihood to estimate $\mathcal{B}_x$ on a weekly level (1) and aggregates the effect on an annual level filtering out seasonal components. (1)

(b) Would you recommend that the insurance company rely on the AG2022 model? Explain your answer carefully.

Solution: The AG2022 model takes COVID-19 into account and is thus more reliable for this specific instrument than the Lee-Carter model (0.5) although there are only two COVID years in the dataset. (0.5) However, the third layer $\mathcal{B}_x\mathcal{X}_t$ is plagued by a lot of uncertainty as the time-effects $\mathcal{X}_t$ are modeled to decay exponentially rather than estimated. (1). Besides, the model assumes (and potentially underestimates) that the COVID-effect is rather transitory, which can lead to a bias in the premium calculation. (0.5) Thus, I would not recommend this model without reservation. (0.5)

(c) Suppose the insurance company still wants to use the AG2022 model until the release of AG2024. The fund manager asks your advice to propose an appropriate simulation approach to simulate assets and liabilities taking into account interest rate risk, micro and macro longevity risk, and stock market risk.

(i) What is different in the law of motions of assets and liabilities compared to a pension fund? Please illustrate both the asset side and the liability side with formulas.

Solution: Pension funds (deaths *reduce* liabilities):

$$\text{assets:} \qquad A_{t+1} = A_t[1 + r_t + \pi(R_T - r_t)] - \sum_{x\in\mathcal{X}} N_{x,t}p_{x,t} \qquad ①$$

$$\text{liabilities:} \qquad L_{k+1} = \sum_{x\in\mathcal{X}} N_{x,t}p_{x,t}a^{BE(t+1)}_{x+1,t+1} \qquad ①$$

Life insurance company (deaths *reduce* assets):

$$\text{assets:} \qquad A_{t+1} = A_t[1 + r_t + \pi(R_T - r_t)] - \sum_{x\in\mathcal{X}} N_{x,t}q_{x,t} \qquad ①$$

$$\text{liabilities:} \qquad L_{k+1} = \sum_{x\in\mathcal{X}} N_{x,t}q_{x,t}a^{BE(t+1)}_{x+1,t+1} \qquad ①$$

(ii) Elaborate on how to simulate relevant payment streams in this model taking the above-mentioned risks into account.

Solution: At $t = 0$, the best-estimate death probability is not uncertain, and thus $N_{x+1,t+1}$ is binomial distributed. ①

Simulate a large number (e.g., 100,000) of random variables drawn from a binomial distribution with parameters $p^{BE}_{x,t}$ and $N_{x,t}$. ①

In the subsequent years, the survival probabilities need to be simulated from the AG2022 model and are then path-dependent as they explicitely depend on the number of survivors in the previous period, which is a random variable itself. Thus, the future survivors are not binomially distributed. ②

Simulate (independently of mortality risk) the same number of interest rate and stock market scenarios for an appropriately chosen stock market model (e.g., Heston) and interest rate model (e.g., Vasicek). Use those simulations to simulate the assets, liabilities, funding ratios, and other variables of interest. ②