Life Insurance
– Assignment: Macro Longevity Risk –

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Introduction

- **Group size**: The assignment should be done in groups of three to five students.

- **Deadline**: 8pm, June 12, 2026.
- **Problems**: There are four exercises. For each exercise you can get 25 points, i.e., 100 points in total. This assignment counts for 15% of the final grade.
- **Data**: Use the data from the website of the Actuarial Association (`https://www.actuarieelgenootschap.nl/kennisbank/projections-life-table-ag2024-1`)
  to download the number of Dutch male and female deaths ($D_{x,t}^{(g)}$) and exposures ($E_{x,t}^{(g)}$) for the years 1970 to 2019 and the ages 0 to 90 years.
- **Programming Language**: Your code, your choice.
- **Submission**: Submit your typesetted results along with your code to o.p.h.labout@tilburguniversity.edu

Problem 1: Lee-Carter Model

The Lee-Carter model is given by

$$\ln(m_{x,t}^{(g)}) = \alpha_x^{(g)} + \beta_x^{(g)} \kappa_t^{(g)} + \varepsilon_{x,t}^{(g)},$$
$$\kappa_t^{(g)} = c^{(g)} + \kappa_{t-1}^{(g)} + \delta_t^{(g)},$$

where $m_{x,t}^{(g)} = \frac{D_{x,t}^{(g)}}{E_{x,t}^{(g)}}$, $\varepsilon_{x,t}^{(g)} \sim_{i.i.d.} \mathcal{N}(0, \sigma^2_{\varepsilon^{(g)}})$, and $\delta_t^{(g)} \sim_{i.i.d.} \mathcal{N}(0, \sigma^2_{\delta^{(g)}})$.

1. Estimate the Lee-Carter model for both genders using the original Lee-Carter approach with (i) singular value decomposition, (ii) itereated least squares.
   - Report the estimation results of $\alpha_x^{(g)}$, $\beta_x^{(g)}$, and $\kappa_t^{(g)}$ in graphs, using the Lee-Carter normalization.
   - Also report the estimates of $c^{(g)}$ and $\sigma_{\delta^{(g)}}$.
2. Show a sequence of graphs that illustrates the convergence of the iterative estimation procedure.
3. Comment on your findings including a comparison of the outcomes of the two estimation approaches.

Problem 2: Cairns-Blake-Dowd (CBD) Model

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The CBD model (Cairns, Blake, Dowd, 2006) specifies the logit of the one-year death probability directly as a linear function of age:

$$\ln\left(\frac{q_{x,t}^{(g)}}{p_{x,t}^{(g)}}\right) = \kappa_{1,t}^{(g)} + \kappa_{2,t}^{(g)}\,(x - \bar{x}),$$

where $\bar{x}$ denotes the mean age in the sample and $(\kappa_{1,t}^{(g)}, \kappa_{2,t}^{(g)})$ follow a bivariate random walk with drift.

1. Explain what happens if you replace the left-hand side of the Lee-Carter model $\ln(m_{x,t}^{(g)})$ by $\ln\left(q_{x,t}^{(g)}/p_{x,t}^{(g)}\right)$ (CBD indicator), and re-estimate the Lee-Carter model with this alternative indicator.
2. Estimate the full CBD model for both genders via OLS and report the time series of $\hat{\kappa}_{1,t}^{(g)}$ and $\hat{\kappa}_{2,t}^{(g)}$ in graphs. Interpret the results economically. Also estimate the bivariate random walk with drift for $(\kappa_{1,t}^{(g)}, \kappa_{2,t}^{(g)})$ and report the estimation results.
3. Compare fitted mortality rates and life expectancies from the CBD model to those from the Lee-Carter model for ages $x = 45, 65, 85$.

Problem 3: High-Age Extrapolation

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We consider two methods to extrapolate mortality rates beyond age 90.

Kannisto method. For ages $x \in \{91, \ldots, 120\}$, $\widehat{\mu}^{(g)}_{x,T+t}$ is determined as

$$\widehat{\mu}^{(g)}_{x,T+t} = L\left(\sum_{k=80}^{90} w_k(x)\, L^{-1}\left(\widehat{\mu}^{(g)}_{k,T+t}\right)\right),$$

where $L(z) = \frac{1}{1+e^{-z}}$, $L^{-1}(z) = \ln\left(\frac{z}{1-z}\right)$, and

$$w_k(x) = \frac{1}{11} + \frac{(k-85)(x-85)}{110}.$$

Coale–Kisker method. For ages $x \in \{91, \ldots, 120\}$, log-mortality is extrapolated linearly from age 80:

$$\ln \widehat{\mu}^{(g)}_{x,t} = \ln \widehat{\mu}^{(g)}_{80,t} + s^{(g)}_t\,(x-80),$$

where the slope $s^{(g)}_t$ is calibrated so that $\widehat{\mu}^{(g)}_{120,t} = 1$ (i.e., certain death by age 120).

Problem 3: High-Age Extrapolation

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1. Determine $\widehat{\mu}_{x,t}^{(g)}$ and $\widehat{p}_{x,t}^{(g)}$ for ages $x = 91, \ldots, 120$ and years $t = 1970, \ldots, 2019$ using both the Kannisto and the Coale–Kisker method, based on your Lee-Carter estimates from Problem 1.
   - Illustrate how the 1-year survival probabilities of 95-year-olds have evolved over time for both males and females under each method.
   - Plot the full age profile of extrapolated mortality rates for selected years (e.g. 1970, 1990, 2019) and both genders, comparing the two methods.
2. Compute the **period remaining life expectancies** in-sample for both males and females for four ages ($x = 5, 35, 65, 95$), using each extrapolation method. Show the outcomes in graphs.
   - Discuss the sensitivity of remaining life expectancy at age 95 to the choice of extrapolation method.
3. Briefly compare the two extrapolation methods.
   - What are the structural assumptions underlying each method?
   - For which age groups and applications would you prefer one method over the other, and why?

1. **Fan charts.** For ages $x \in \{35, 65, 80\}$ and both genders, compute the distribution of remaining period life expectancy across the S simulated trajectories for each horizon year $T + s$, $s = 1, \ldots, 20$.
   - Present fan charts showing the 10th, 25th, 50th, 75th, and 90th percentiles of remaining life expectancy over the 20-year horizon.
2. **Comparison with AG2024.** Download the AG2024 projected mortality rates and compute:
   - The difference in remaining life expectancy (your Lee-Carter median vs. AG2024 central projection) for $x \in \{35, 65, 80\}$ over the 20-year horizon.
   - The fraction of your simulated trajectories that lie above the AG2024 central projection.
   - Comment on the main sources of discrepancy (model specification, parameter uncertainty, extrapolation) and their relevance for practice.