The Dismal Theorem in a General Equilibrium Climate-Economy Model

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Keywords: Social cost of carbon, dismal theorem, fat tails, recursive preferences, climate disasters, general equilibrium

JEL subject codes: Q54, G12, D81

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1 Introduction

A cornerstone of climate economics is the social cost of carbon (SCC), defined as the expected present discounted value of all current and future damages from emitting one additional ton of carbon. A challenge in computing the SCC is extreme climate risk. Weitzman (2009) shows that under fat-tailed uncertainty about climate damages and constant relative risk aversion (CRRA) utility, expected utility and expected marginal utility may be infinite, rendering cost-benefit analysis inapplicable. This is known as the *dismal theorem*. It applies if the probability of catastrophic outcomes decreases more slowly than the CRRA marginal utility weight grows, so the welfare integral diverges.

Geweke (2001) already showed that the existence of expected utility under CRRA is extremely fragile with respect to distributional assumptions: if the endowment is log-normal with uncertain mean growth rate, expected utility fails to exist except for for log-utility. Nordhaus (2011) argues that the conditions required for the theorem to hold limit its practical relevance and that there is no universal rule for determining when cost-benefit analysis breaks down. Pindyck (2011) draws a conceptual distinction between fat-tailed *distributions* for outcomes and fat-tailed *utility functions*. It is unbounded marginal utility as consumption approaches zero, not distributional fat-tailedness alone, that drives the dismal result. Once marginal utility is bounded, the expected SCC remains finite even under fat-tailed outcomes. Millner (2013) provides the most systematic analytical treatment, classifying objections to the dismal theorem into three categories and finding that critiques based on the utility function specification are the most robust.[1] Anthoff and Tol (2022) test the dismal theorem empirically by estimating the tail index of published SCC distributions and find mixed evidence across integrated assessment models.

None of these contributions embed the dismal mechanism in a fully optimizing general equilibrium production economy with recursive preferences, endogenous growth, temperature-dependent disaster risk, and a growth rate impact of warming on capital accumulation. This note fills that gap. We derive an *exact* expression for the optimal SCC and show that the dismal mechanism resurfaces within this framework if the loss distribution from climate-related disasters is sufficiently fat-tailed: the risk-adjusted expected loss term in the SCC then diverges, and the model has no finite solution. We further show that the dismal mechanism can be activated by a mixture beta distribution that is empirically indistinguishable from the calibration based on a power distribution used in van den Bremer et al. (2026) at the level of first and second moments, underscoring the empirical relevance of the result. Finally, we discuss that if the expected size of damages also increases with temperature, the fat tail becomes endogenous. The dismal mechanism is then more likely to be activated at higher temperatures.

2 Model

We build on the model of van den Bremer et al. (2026) but restrict attention to a unit elasticity of intertemporal substitution to obtain exact closed-form solutions that are valid without approximation and to ensure that our findings are not compromised by their perturbation approximation. The aggregate capital stock K_t evolves as

$$\mathrm{d}K_t = \left(I_t - \delta K_t - \frac{1}{2}\phi \frac{I_t^2}{K_t} - \xi T_t K_t\right)\mathrm{d}t + K_t \sigma\,\mathrm{d}W_t - K_{t-}\ell\,\mathrm{d}N_t, \tag{2.1}$$

where I_t denotes aggregate investment, $\delta \geqslant 0$ the depreciation rate, ϕ the investment adjustment cost parameter, and W_t a Wiener process with $\sigma \geqslant 0$ the volatility of capital. The parameter $\xi \geqslant 0$ captures the *growth rate impact* of global warming as in Dell et al. (2012): higher temperature T_t reduces the effective return on capital, permanently lowering economic growth. The Poisson process N_t captures climate-related disasters with temperature-dependent intensity $\lambda(T_t) = \lambda_0 + \lambda_1 T_t$ and random loss fraction $\ell \in (0,1)$. The recovery rate is $R = 1 - \ell$.

The final goods production function is $Y_t = AK_t^{\alpha}E_t^{1-\alpha}$, where A is total factor productivity and $\alpha \in (0,1)$. $E_t = \left(\kappa F_t^{\eta} + (1-\kappa)G_t^{\eta}\right)^{1/\eta}$ is an energy composite consisting of fossil fuel use F_t and

[1] He further proposes an alternative welfare framework drawing on population ethics to handle catastrophes involving changes in population size.

green energy G_t. The budget constraint is $Y_t = I_t + C_t + b_f F_t + b_g G_t$, where b_f and b_g are the unit costs of fossil fuel and green energy, respectively (Hambel and van der Ploeg, 2025).

Emissions are proportional to fossil-fuel use, and temperature is proportional to cumulative emissions (Allen et al., 2009), so

$$\mathrm{d}T_t = \chi \varpi_t F_t \mathrm{d}t, \tag{2.2}$$

where $\chi > 0$ denotes the transient climate response to cumulative emissions (TCRE) and ϖ_t the emission intensity, declining at the rate of economic growth. This linear relationship between temperature and cumulative emissions avoids the excessive inertia in standard integrated assessment models (Dietz et al., 2021; Folini et al., 2025).

The continuous-time Epstein-Zin aggregator for unit EIS is (Duffie and Epstein, 1992)

$$\mathscr{F}(C_t, J_t) = \rho J \log\left(\frac{C_t}{[(1-\gamma)J_t]^{1/(1-\gamma)}}\right), \tag{2.3}$$

where $\rho \geqslant 0$ denotes the pure rate of time preference and $\gamma > 0$ the coefficient of relative risk aversion (RRA). The representative agent's value function is thus recursively defined as $J_t = \sup_{(C_s,F_s)_{s\geqslant t}} \mathbb{E}_t\left[\int_t^\infty \mathscr{F}(C_s, J_s)\,\mathrm{d}s\right]$. The following result for social cost of carbon (SCC) or optimal carbon price is shown in Appendix A.

Result 2.1 (Optimal Carbon Price)**.** *The optimal carbon price is*

$$P_t = \frac{\chi}{\rho}\left(\xi + \lambda_1 \frac{\mathbb{E}[R^{1-\gamma} - 1]}{\gamma - 1}\right) K_t q, \tag{2.4}$$

where $\lambda_1 = \partial\lambda/\partial T$, $q = 1/(1-\phi i)$ is Tobin's Q, and $i = I_t/K_t$ the equilibrium investment rate.

The SCC is thus the present discounted value of the marginal growth-rate damages and expected risk-adjusted marginal damages from disasters to the capital stock, where the growth- and risk-adjusted discount rate boils down to the pure rate of time preference ρ.[2]

3 The Dismal Condition

This section examines the optimal carbon price (2.4) when the climate disaster loss distribution is sufficiently fat-tailed, and analyzes the dismal result.

3.1 Main Result

While the growth rate impact affects the SCC in a linear manner through ξ, the key object in our analysis is the moment $\mathscr{M} = \mathbb{E}[R^{1-\gamma}] > 0$, which enters the SCC in (2.4) through the

[2] If the EIS differs from one, this discount rate would depend on the expected growth rate and the characteristics of the stochastic shocks van den Bremer et al. (2026).

disaster risk channel and depends non-linearly on RRA γ. Since $\gamma > 1$, we have $R^{1-\gamma} \to \infty$ as $R \to 0$. The moment $\mathscr{M}$ is therefore finite if and only if the density f of R, denoted by $f(r)$, is sufficiently thin near zero.

Definition 3.1 (Fat-tailed loss distribution)**.** *The climate disaster loss distribution is* fat-tailed *if*

$$\int_0^1 r^{1-\gamma} f(r)\,\mathrm{d}r = \infty, \tag{3.1}$$

i.e., the moment $\mathscr{M} = \mathbb{E}[R^{1-\gamma}]$ does not exist.

The definition places no restrictions on the functional form of f and does not require a positive mass at $\ell = 1$: the integral in (3.1) diverges whenever f near zero decays more slowly than $R^{\gamma-1}$, which is a pure tail-index condition. The well-definedness of the SCC does not imply that the model is otherwise well-posed under a fat-tailed loss distribution: one can show that equilibrium asset returns involve moments of the form $\mathbb{E}[R^{-\gamma}]$, which diverge under the weaker condition $\int_0^1 r^{-\gamma} f(r)\,\mathrm{d}r = \infty$ (e.g., Pindyck and Wang, 2013; van den Bremer et al., 2026).

Result 3.2 (Dismal Condition)**.** *The optimal carbon price* (2.4) *is finite if and only if the moment $\mathscr{M} = \mathbb{E}[R^{1-\gamma}]$ is finite. If the climate disaster loss distribution is fat-tailed, it is infinite, and it is optimal to switch immediately to renewable energy.*

The disaster intensity $\lambda(T)$ rises with temperature, but this does not affect the dismal condition (3.2), which depends solely on the shape of $f(r)$ near $r = 0$.

3.2 Two Illustrative Distributions

We illustrate Result 3.2 with two concrete loss distributions that match the empirically observed expected climate disaster impact of 1.58% of output (Karydas and Xepapadeas, 2022). These distributions are shown in Figure 1.

Power distribution with thin tails For the widely-used standard power distribution with shape parameter β and density $f(r) = \beta r^{\beta-1}$ on $(0,1)$ (Barro and Jin, 2011; Pindyck and Wang, 2013; van den Bremer et al., 2026). The moment $\mathscr{M} = \beta/(\beta+1-\gamma)$ exists if and only if $\beta > \gamma - 1$; else the dismal theorem applies. Since van den Bremer et al. (2026) calibrate $\beta = 62.29$ to match the expected climate disaster size of $\mathbb{E}[\ell] = 1/(1+\beta) = 1.58\%$, the dismal theorem only applies for unrealistically high degrees of risk aversion: $\gamma > 1 + \beta = 63.29$). Hence, typically, the moment $\mathscr{M}$ is finite and the SCC is well-defined.

The distribution is strongly concentrated near $r = 1$ with a standard deviation of $\sigma[\ell] = 1.56\%$, reflecting the empirical regularity that most climate disasters cause moderate capital losses, see Figure 1. Activating the dismal mechanism either requires an unrealistically high γ or an unrealistic high expected loss $\mathbb{E}[\ell]$: $\gamma\mathbb{E}[\ell] \geqslant 1$. Even for a high RRA ($\gamma = 10$), this requires a shape parameter of at most $\beta = 9$, implying an expected loss of at least 10% to activate the

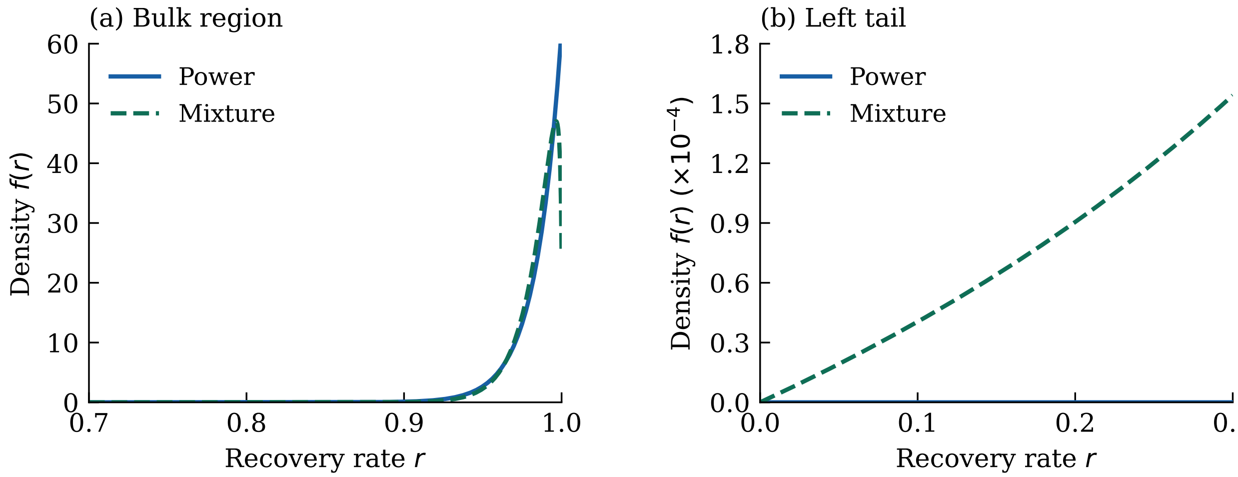

Figure 1: Density functions of the two loss distributions. Panel (a) shows the right bulk for $r \geqslant 0.7$ and Panel (b) shows the left tail for $r \leqslant 0.1$. While the left tail of the power distribution is practically zero ($f(r) \leqslant 10^{-49}$ for $r < 0.15$), the left tail of the mixture distribution carries enough weight to activate the dismal mechanism even though the density is very small.

dismal mechanism (see the blue line in Figure 2). This is far outside the empirical estimates (e.g., Karydas and Xepapadeas, 2022). Therefore, the power distribution cannot activate the dismal mechanism without shifting the expected loss to much higher levels than currently empirically observed.

Beta mixture distribution with fat tails To show that the dismal mechanism can be activated *without* altering the expected loss when a disaster strikes, let R follow a two-component beta mixture distribution

$$R \sim \pi \operatorname{Beta}(a_1, b_1) + (1-\pi) \operatorname{Beta}(a_2, b_2), \tag{3.2}$$

so with probability $\pi \in [0,1]$ the recovery rate is drawn from $\operatorname{Beta}(a_1, b_1)$ and with probability $1-\pi$ from $\operatorname{Beta}(a_2, b_2)$. The density of R is

$$f(r) = \pi \frac{r^{a_1-1}(1-r)^{b_1-1}}{B(a_1,b_1)} + (1-\pi) \frac{r^{a_2-1}(1-r)^{b_2-1}}{B(a_2,b_2)}, \qquad r \in (0,1), \tag{3.3}$$

where $B(a,b) = \int_0^1 z^{a-1}(1-z)^{b-1}\,\mathrm{d}z$ is the beta function. We refer to $\operatorname{Beta}(a_1,b_1)$ as the *bulk component* and to $\operatorname{Beta}(a_2,b_2)$ as the *tail component* of the mixture distribution. Notice that for a $\operatorname{Beta}(a,b)$ distribution, $\int_0^1 r^{1-\gamma} f(r)\,\mathrm{d}r = B(a+1-\gamma,b)/B(a,b)$, which diverges whenever $a \leqslant \gamma - 1$.

To illustrate the effects of this distribution, we calibrate the parameters so that they match the first two moments of the power distribution above: $\mathbb{E}[\ell] = 1.58\%$ and $\sigma[\ell] = 1.56\%$. We chose $a_1 = 80$, $b_1 = 1.28$ so the mean loss of the bulk component is 1.57%. We further set $a_2 = 2$,

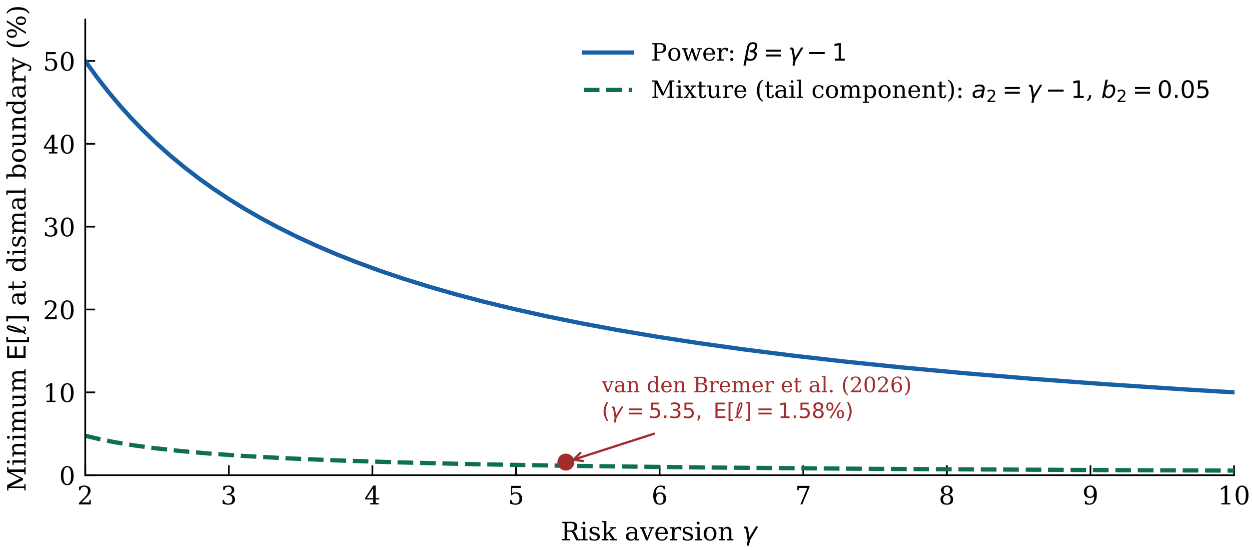

Figure 2: Minimal expected loss to activate the dismal result. The graph plots the minimal expected loss $\mathbb{E}[\ell]$ required to activate the dismal mechanism as function of the coefficient of relative risk aversion γ. The solid blue line shows the boundary for the power distribution, $\beta = \gamma - 1$, at which $\mathbb{E}[\ell] = 1/\gamma$. The dashed green line shows the boundary for tail component of the mixture distribution with $b_2 = 0.05$ fixed, $a_2 = \gamma - 1$, at which $\mathbb{E}[\ell] = 0.05/(\gamma - 0.95)$. Above each curve the dismal condition is satisfied. The red dot marks the van den Bremer et al. (2026) calibration ($\gamma = 5.35$, $\mathbb{E}[\ell] = 1.58\%$), just above the beta mixture boundary but far below the power boundary, illustrating that the beta mixture can activate the dismal mechanism at empirically plausible loss sizes while a power distribution cannot.

$b_2 = 0.05$, so the tail component alone has infinite $\mathscr{M}$ since $a_2 = 2 < \gamma - 1$ for all $\gamma > 3$. For any mixture weight $\pi > 0$, the full mixture also has infinite $\mathscr{M}$ if $\gamma > 3$, regardless of $\pi \neq 1$. Setting $\pi = 99.3\%$ yields $\mathbb{E}[\ell] = 1.58\%$ and $\sigma[\ell] = 1.56\%$, matching the first two moments of the power distribution model.

Comparison The density function of the mixture distribution behaves very similarly to the van den Bremer et al. (2026) calibration in the bulk (Panel a of Figure 1): the overwhelming majority of disaster realizations are mild. Across the bulk of the distribution, which spans losses from 0.17% to 3.6% and thus covers the empirically relevant range, quantiles differ by at most 0.11 percentage points. The difference only becomes apparent in the extreme left tail (Panel b): the mixture assigns a thin but mathematically critical left tail through the $\mathrm{Beta}(2, 0.05)$ tail component. Only beyond losses of 25% do the distributions diverge radically, with the mixture assigning probability of the order 10^{-4} to near-total capital destruction while the power distribution assigns essentially zero. This is precisely the region of the loss distribution that activates the dismal mechanism. Since the integrand $r^{1-\gamma}$ diverges as $r \to 0$ with $\gamma > 1$, even a tail weight of only 0.7% is sufficient to make $\mathscr{M}$ infinite.

The expected loss $\mathbb{E}[\ell] = 1.58\%$ as used as calibration target by van den Bremer et al. (2026) cannot distinguish between the power and mixture distributions. Even taking the second moment into account is not sufficient to detect the dismal mechanism, which requires estimating the tail behavior of $f(r)$ near $r = 0$, i.e., the region where observed data on climate disaster losses are scarcest. This echoes Weitzman (2009)'s original argument that fat-tail behavior can never be fully learned away from finite data.

4 Comparison to Weitzman (2009)'s Dismal Theorem

Result 3.2 is the dynamic general equilibrium analogue of Weitzman (2009)'s dismal theorem. Three features distinguish our result.

(i) General Equilibrium. All prior contributions work in reduced-form or partial equilibrium economies. Our result is embedded in a fully optimizing general equilibrium production economy with endogenous growth and an explicit carbon pricing rule. The diverging term $\mathcal{M} = \mathbb{E}[R^{1-\gamma}]$ is the risk-adjusted expected utility loss from a climate disaster, i.e., the general equilibrium counterpart of Weitzman's divergent marginal utility integral.

(ii) Recursive Preferences. Weitzman's result is framed in terms of CRRA utility, which links risk aversion and the EIS. Recursive preferences decouple these two parameters. At unit EIS, the denominator of the SCC equals ρ regardless of the loss distribution, so the diverging numerator cannot be offset. This provides a sharper characterization than is available in the reduced-form literature, and connects to Geweke (2001)'s observation that unit EIS is a knife-edge case for the existence of expected utility under CRRA with uncertain growth rates.

(iii) Endogenous Fat Tail. In Weitzman's framework, the fat tail of the damage distribution is an assumption about structural uncertainty regarding climate sensitivity. In our setting, the fat-tail condition (3.1) concerns the loss distribution of climate disasters. The disaster intensity $\lambda(T_t)$ rises with temperature but does not affect the condition (3.2) for the dismal theorem to hold, which depends solely on the shape of $f(r)$ near $r = 0$. However, we conjecture that if the expected disaster losses increase with global warming, and thus the parameter β falls with global warming, the dismal theorem is more likely to be activated at higher temperatures.

Hence, if a climate tipping point shifts the loss distribution from a thin-tailed to a fat-tailed regime, the pre-tipping SCC already diverges because the economy faces a positive probability of transitioning to a state where $\mathcal{M} = \infty$. Since the disaster probability rises with temperature, the dismal mechanism is in this sense activated by carbon accumulation. We leave a formal treatment for future research.

5 Conclusion

We have derived a closed-form expression for the SCC in a general equilibrium climate-economy model with Epstein-Zin preferences, endogenous growth, climate-related disaster risk, and a growth rate impact of warming. We show that Weitzman's dismal theorem resurfaces whenever the climate disaster loss distribution is fat-tailed in the sense of Definition 3.1. When

$\mathbb{E}[R^{1-\gamma}-1]/(\gamma-1)$ is infinite, the SCC is infinite and it becomes optimal to abolish fossil fuel immediately. While the calibration of van den Bremer et al. (2026) implies finite moments, the dismal condition can be activated by loss distributions that are empirically indistinguishable at the first and second moments.

For small island states, near-total capital destruction is not merely theoretical: Hurricane Maria caused damage to Dominica in 2017 estimated at 225% of its annual GDP, and the combined impact of Cyclone Harold and COVID-19 caused losses equivalent to approximately 61% of Vanuatu's GDP in 2020 (UNESCAP, 2021; UNDESA, 2021), illustrating that climate-induced disasters can cause damage equivalent to a multiple of annual output and the dismal result to hold.

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A Solution of the Model

The value function satisfies the following Hamilton-Jacobi-Bellman (HJB) equation:

$$0 = \sup_{F,G,C} \Big\{ \mathscr{F}(C,J) + J_K \Big(I - \frac{1}{2}\phi I^2 - \delta K - \xi T K \Big) + \frac{1}{2} J_{KK} K^2 \sigma^2 \\ + J_T \chi \varpi F + \lambda(T) \mathbb{E}\big[J(K(1-\ell),T) - J(K,T) \big] \Big\}$$

where $I = AK^\alpha E(F,G)^{1-\alpha} - C - b_f F - b_g G$, and ϖ is proportional to $1/K$. We define the relative controls $i = I/K$, $c = C/K$, $f = F/K$, and $g = G/K$. We conjecture the following value function

$$J(K,T) = \frac{1}{1-\gamma} K^{1-\gamma} V(T) \tag{A.1}$$

for an unknown function $V(T)$. We substitute the conjecture into the HJB equation and obtain the following HJB equation for V,

$$0 = \sup_{f,g,c} \Big\{ \rho(1-\gamma)\log(c)V - \rho V \log(V) + (1-\gamma)\big(i - 0.5\phi i^2 - \delta - \xi T\big)V - \frac{1}{2}\gamma(1-\gamma)\sigma^2 V \\ + V_T \chi f + \lambda(T)\mathbb{E}\big[(1-\ell)^{1-\gamma} - 1\big]V \Big\},$$

where investments are $i = A\big(\kappa f^\eta + (1-\kappa) g^\eta\big)^{(1-\alpha)/\eta} - c - b_f f - b_g g$. With linear climate disaster intensity $\lambda(T) = \lambda_0 + \lambda_1 T$, we conjecture the following functional form

$$V(T) = \mathrm{e}^{\omega_0 + \omega_1 T}.$$

Consequently, the HJB equation can be written as

$$0 = \sup_{f,g,c} \Big\{ \rho(1-\gamma)\log(c) - \rho(\omega_0 + \omega_1 T) + (1-\gamma)\big(i - 0.5\phi i^2 - \delta - \xi T\big) - \frac{1}{2}\gamma(1-\gamma)\sigma^2 \\ + \omega_1 \chi f + (\lambda_0 + \lambda_1 T)\mathbb{E}\big[(1-\ell)^{1-\gamma} - 1\big] \Big\}.$$

Collecting all terms containing T identifies an equation for ω_1 with solution

$$\omega_1 = \frac{\gamma - 1}{\rho}\Big(\xi + \lambda_1 \frac{\mathbb{E}\big[(1-\ell)^{1-\gamma} - 1\big]}{\gamma - 1} \Big).$$

Consequently, we obtain

$$\omega_0 = \frac{1}{\rho} \sup_{f,g,c} \Big\{ \rho(1-\gamma)\log(c) + (1-\gamma)\big(i - 0.5\phi i^2 - \delta\big) - \frac{1}{2}\gamma(1-\gamma)\sigma^2 + \omega_1 \chi f \\ + \lambda_0 \mathbb{E}\big[(1-\ell)^{1-\gamma} - 1\big] \Big\},$$

which does not depend on temperature. The social planner sets the carbon price P_t equal to the SCC, defined as $SCC_t = -\frac{J_S}{f_C} > 0$, where J_S is the marginal value of cumulative emissions and f_C is the marginal utility of consumption. Since $T_t = \chi S_t$, the SCC is $SCC_t = -\chi\frac{J_T}{f_C}$. The optimal fossil fuel use satisfies $\frac{\partial Y_t}{\partial F_t} = b_f + \varpi_t P_t$. This implies the following optimal carbon price:

$$P_t = SCC_t = \frac{\chi}{\rho}\left(\xi + \frac{\partial\lambda}{\partial T}\frac{\mathbb{E}[R^{1-\gamma}-1]}{\gamma-1}\right)K_t q(i)$$

where $q(i) = 1/(1-\phi i)$ is Tobin's Q. Calculating the first-order conditions yields the following system for the normalized optimal controls:

$$\begin{aligned}
c &= \frac{\rho}{1-\varphi i},\\
i &= A\varepsilon(f,g)^{\frac{1-\alpha}{\eta}} - c - b_f f - b_g g,\\
g &= \left[\frac{b_g}{A(1-\alpha)(1-\kappa)\varepsilon(f,g)^{\frac{1-\alpha}{\eta}-1}}\right]^{\frac{1}{\eta-1}},\\
f &= \left[\frac{b_f+\tau}{A(1-\alpha)\kappa\varepsilon(f,g)^{\frac{1-\alpha}{\eta}-1}}\right]^{\frac{1}{\eta-1}},
\end{aligned}$$

where $\varepsilon(f,g) = \kappa f^\eta + (1-\kappa)g^\eta$ and $\tau = \frac{\chi}{\rho}\left(\xi + \frac{\partial\lambda}{\partial T}\frac{\mathbb{E}[R^{1-\gamma}-1]}{\gamma-1}\right)q(i)$. □