Life Insurance
– Lecture Parts III and IV –

Christoph Hambel

Tilburg University
Tilburg School of Economics and Management
Department of Econometrics and Operations Research

Spring Term 2023

Course Information

- Lecturers:
  - Feiko Drost (I: micro longevity risk and II: interest rate risk)
  - Christoph Hambel (III: macro longevity risk and IV: all risks combined)
  - Henk Keffert (tutorials)

- The second half of this course ...
  - ... provides an introduction to macro longevity risk and to applications in actuarial science that combine all types of risk.
  - ... directly builds upon the first half and does not require any additional pre-knowledge.

- Grading:
  - Exam 70%
  - Two Assignments (15% each)

What to expect?

- What can you expect from me? I will...
  - ... timely provide the learning material on Canvas
  - ... also upload the slides with hand-written complements (some parts of the slides are intentionally blank)
  - ... illustrate the lecture by examples
  - ... provide a lot of problems to practice the material
  - ... be available for questions
- What will I expect from you? You should ...
  - ... be well-prepared when you come to the lecture
  - ... actively participate in the lecture
  - ... take the opportunity and ask me questions during the classes

Please notice that the plan can change!

- Tue, 11.04.2023, 08:45, WZ105
- Tue, 18.04.2023, 08:45, WZ105
- Wed, 19.04.2023, 08:45, CUBE 221 (tutorial)
- Tue, 25.04.2023, 08:45, WZ105
- Wed, 26.04.2023, 08:45, CUBE 221 (tutorial)
- Tue, 09.05.2023, 08:45, WZ105
- Thu, 11.05.2023, 12:45, CZ05
- Tue, 16.05.2023, 08:45, WZ105 (tutorial)
- Wed, 17.05.2023, 16:45, CUBE 218
- Tue, 23.05.2023, 08:45, WZ105
- Wed, 24.05.2023, 08:45, CUBE 221 (tutorial)

Structure of the Course (Second Half)

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Part III: Macro Longevity Risk

1. Introduction
2. Relevance of Macro Longevity Risk
    - First Pillar: AOW
    - Second Pillar: Pension Funds
3. Modeling Mortality
4. Benchmark Model
    - The Lee-Carter Model
    - Alternative Estimation
    - Some Applications and Extensions
5. The AG2022 Model and COVID-19
    - Model and Projections
    - Closure of the Life Table
6. Model Risk: A Very Brief Introduction

Structure of the Course (Second Half)

Part IV: Pricing under all Types of Risk

7. Setting
8. Illustrations
   - No risk
   - Micro longevity risk
   - Macro longevity risk
   - Interest rate risk
   - All risks combined

Part III

Macro Longevity Risk

Table of Contents

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- **Micro Longevity Risk**

  Risk because (for given death probabilities) an individual's *remaining lifetime* is unknown.

  The remaining lifetime of an individual of age x belonging to a group g at time t is modeled as a random variable conditional on the future death probabilities $q^{(g)}_{x+s,t+s}$, $s = 0, 1, 2, \ldots$

- **Macro Longevity Risk**

  Additional risk because future death probabilities are unknown.

  The future death probabilities $q^{(g)}_{x+s,t+s}$, $s = 0, 1, 2, \ldots$, will be modeled as random variables on date t.

The life table for a given group g can be represented as

$$\begin{array}{c} \rightarrow \text{time} \\ \downarrow \\ \text{age} \end{array} \left(\begin{array}{cccc:c} q^{(g)}_{0,t_0} & q^{(g)}_{0,t_0+1} & \cdots & q^{(g)}_{0,t_0+T} & \cdots \\ q^{(g)}_{1,t_0} & q^{(g)}_{1,t_0+1} & \cdots & q^{(g)}_{1,t_0+T} & \cdots \\ q^{(g)}_{2,t_0} & q^{(g)}_{2,t_0+1} & \cdots & q^{(g)}_{2,t_0+T} & \cdots \\ \vdots & & & \vdots & \\ \vdots & & & \vdots & \\ q^{(g)}_{x_{\max},t_0} & q^{(g)}_{x_{\max},t_0+1} & \cdots & q^{(g)}_{x_{\max},t_0+T} & \cdots \end{array} \right) \begin{array}{l} \rightarrow \textit{Predictions} \\ \rightarrow \textit{Predictions} \\ \rightarrow \textit{Predictions} \\ \\ \\ \rightarrow \textit{Predictions} \end{array}$$

AG2022: $x_{\max=120}$, observed: $t_0 = 1970$, $t_0 + T = 2021$, predicted: $t_0 + T + s \geq 2022$

Life Table of Group g: 2 Questions

1. How to estimate/calibrate the observed part?
2. How to determine the predictions and the uncertainty surrounding these predictions (macro longevity risk)?

$$\begin{array}{c} \\ \downarrow \\ \text{age} \end{array}
\overset{\rightarrow \text{ time}}{\left(\begin{array}{cccc:c}
q^{(g)}_{0,t_0} & q^{(g)}_{0,t_0+1} & \cdots & q^{(g)}_{0,t_0+T} & \cdots \\
q^{(g)}_{1,t_0} & q^{(g)}_{1,t_0+1} & \cdots & q^{(g)}_{1,t_0+T} & \cdots \\
q^{(g)}_{2,t_0} & q^{(g)}_{2,t_0+1} & \cdots & q^{(g)}_{2,t_0+T} & \cdots \\
\vdots & & & \vdots & \\
\vdots & & & \vdots & \\
q^{(g)}_{x_{\max},t_0} & q^{(g)}_{x_{\max},t_0+1} & \cdots & q^{(g)}_{x_{\max},t_0+T} & \cdots
\end{array}\right)}
\begin{array}{l} \rightarrow \textit{Predictions} \\ \rightarrow \textit{Predictions} \\ \rightarrow \textit{Predictions} \\ \\ \\ \rightarrow \textit{Predictions} \end{array}$$

Period Calculations and Cohort Calculations

- **Period Calculations**
  - Period calculations is using the columns (e.g., copy the final column) of a life table to predict the next period death probability.
  - This means that any future changes to mortality rates would not be taken into account.
  - Period life expectancies use mortality rates from a single year and assume that those rates apply throughout the remainder of a person's life.

- **Cohort Calulations**
  - Cohort calculations is taking future trends into account using models.
  - A cohort life table uses a combination of observed mortality rates for the cohort for past years and projections about mortality rates for the cohort for future years.
  - Requires a model.

→ Period life expectancy would match cohort life expectancy only if there were no changes in age-specific mortality rates over time.

Traditional Approach: Naive Forecast

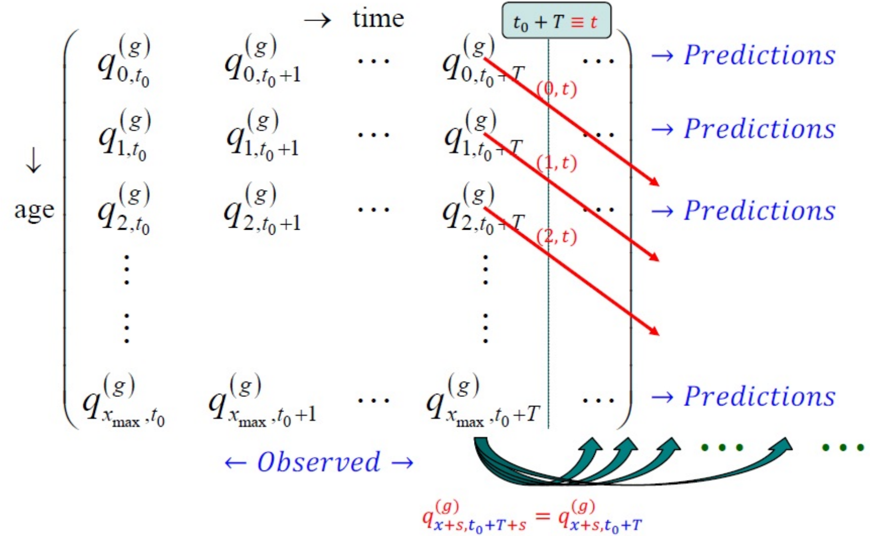
- Traditionally, macro longevity risk was ignored.
- One assumed that the most recently estimated period death probabilities hold true for all future years, i.e., for the cohort $(x, t_0 + T)$ one assumed

$$\underbrace{q^{(g)}_{x+s,t_0+T+s}}_{\textit{predicted}} = \underbrace{q^{(g)}_{x+s,t_0+T}}_{\textit{observed}}$$

  for all $s \geq 0$ and all ages x.
- This means that – if we ignore macro longevity risk – the entries in the last column of the observation part of the life table equal the entries of the prediction part.

Traditional Approach: Naive Forecast

Traditional Approach: Naive Forecast



Period vs. Cohort Calculations

Both period and cohort calculations have some drawbacks:

- **Drawbacks of period calculations**
  - Ignoring trends in death probabilities may lead to significant overestimation of death probabilities.
  - Ignoring uncertainty in future death probabilities may lead to significant underestimation of the risk in life insurance portfolios.
  - Sensitive to (transitory) shocks, e.g., WW2, Spanish flu, COVID-19.
- **Drawbacks of cohort calculations**
  - We unavoidably introduce model risk if we use forecasts.

Current Practice

- Statistics Netherlands (CBS) and the Royal Dutch Actuarial Association (AG) produce point forecasts for future one-year death probabilities by age and gender.

  $\rightarrow$ Are available on the website of the AG.

- These point forecasts are referred to as *best-estimate death probabilities*.

- The AG-models also easily allow the quantification of (at least part of the) macro longevity risk.

- To mitigate the effect of model risk, these best-estimate death probability forecasts are revised annually (CBS, December), or bi-annually (AG, September in even years).

Recall: Some Formulas for Cohort (x, t)

- **τ-years-from-now survival probability:**

$$ {}_\tau p_{x,t}^{(g)} = \prod_{k=0}^{\tau-1} p_{x+k,t+k}^{(g)}, \qquad p_{x,t}^{(g)} = 1 - q_{x,t}^{(g)} $$

- **Remaining life expectancy:**

$$ e_{x,t}^{(g)} = \sum_{\tau=1}^{\infty} {}_\tau p_{x,t}^{(g)} + 0.5 $$

- **Value of immediate single life annuity:**

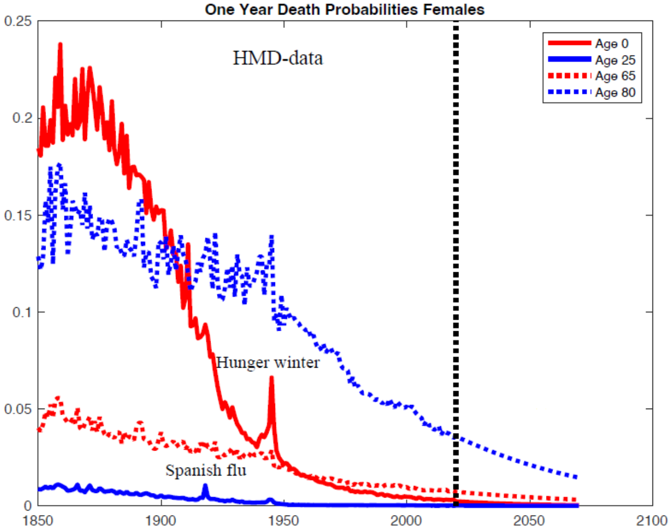
$$ a_{x,t}^{(g)} = \sum_{\tau=1}^{\infty} {}_\tau p_{x,t}^{(g)} \frac{1}{\big(1 + R_t(t+\tau)\big)^{\tau}} $$

- **Value of T-years deferred single life annuity:**

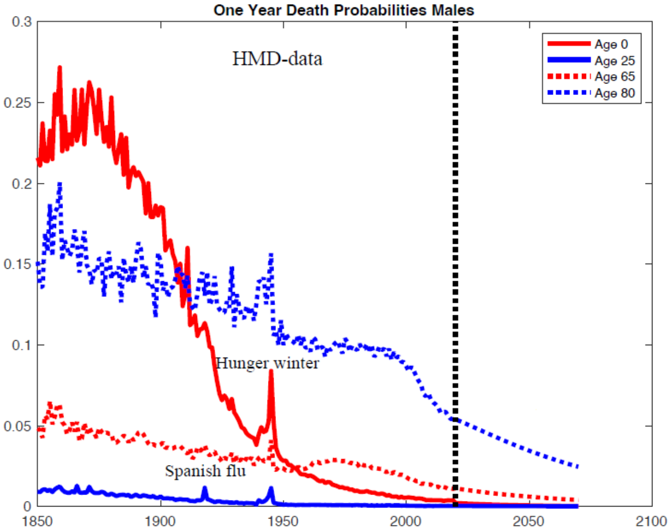
$$ a_{x,t}^{(g)}(T) = \sum_{\tau=T}^{\infty} {}_\tau p_{x,t}^{(g)} \frac{1}{\big(1 + R_t(t+\tau)\big)^{\tau}} $$

Recall: Some Formulas for Cohort (x, t)

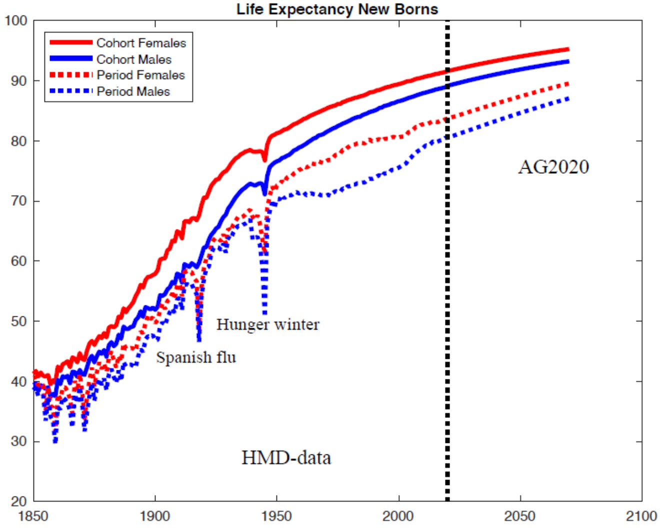
One-Year Death Probabilities



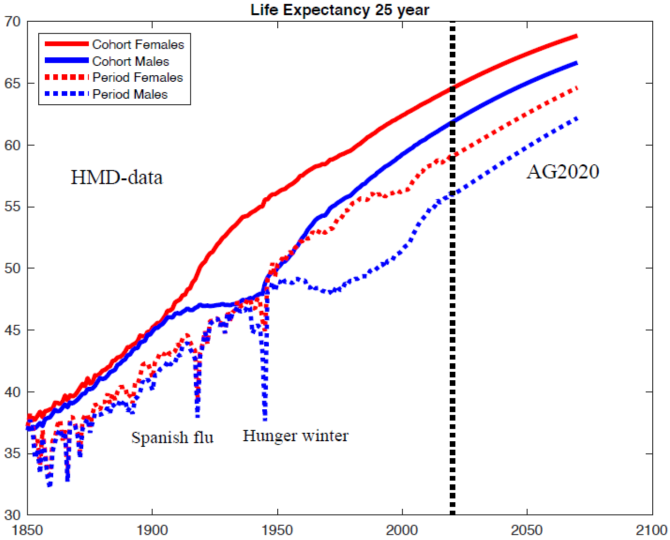
One-Year Death Probabilities



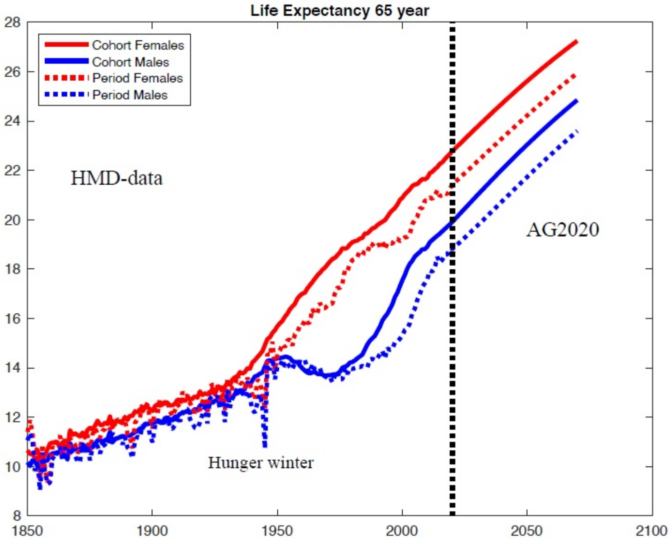
Remaining Life Expectancy (Newborns)



Remaining Life Expectancy (Age 25)



Remaining Life Expectancy (Age 65)



Remaining Life Expectancy (Age 80)