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Exercises to the lecture

"Advanced Financial Economics I"

Problem Set 2

Due: 30 November 2021, 14.15 pm

Please submit your solution via email to christoph.hambel@hof.uni-frankfurt.de or upload it into the OLAT folder.

Problem 1 (Recursive Utility and Asset Pricing)

Consider a pure-exchange economy with infinite time horizon. At time t, the representative household chooses the level of consumption to maximize its utility index $\mathcal{U}_t$ that satisfies

$$\mathcal{U}_t = \left[C_t^{1-1/\psi} + \beta\Big(\mathbb{E}_t\Big[\mathcal{U}_{t+1}^{1-\gamma}\Big]\Big)^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}}.$$

with $\psi \geq 0$, $\gamma \geq 0$, $\beta \in (0,1]$. C_t denotes the representative household's consumption in period t. Suppose that the economy is in general equilibrium and the representative household's log-consumption evolves according to

$$\Delta c_{t+1} = \mu + \sigma\eta_{c,t+1} + \nu_{t+1}$$

where $\mu, \sigma \in \mathbb{R}$, $\eta_{c,t+1} \sim \mathcal{N}(0,1)$, i.i.d. and ν_{t+1} satisfies

$$\mathbb{P}\big(\nu_{t+1} = \log(1-b_{t+1})\big) = p, \qquad \mathbb{P}(\nu_{t+1} = 0) = 1-p$$

where b_{t+1} is i.i.d. with mean $\bar{b} \geq 0$.

1. Show that the indirect utility function has the form

$$J_t = \Phi C_t.$$

for a constant Φ to be determined.

2. Write down the Euler equation for the price of an arbitrary asset.

3. Show that the consumption-wealth ratio is constant and find an expression for it in terms of γ, ψ, and Φ.

4. Determine the risk-free interest rate and the equity premium in this economy.

5. Interpret your results form part 4. and explain why this setting might be able to resolve the equity premium puzzle and the risk-free rate puzzle.

6. By contrast to Bansal and Yaron (2004) "*Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles*" the present model generates constant asset pricing moments. Assume now that the household's log-consumption evolves according to

$$\Delta c_{t+1} = \mu + y_t + \sigma \eta_{c,t+1} + \nu_{t+1}$$
$$\Delta y_{t+1} = \rho y_t + \phi_y \eta_{y,t+1}$$

where $\rho, \kappa \in \mathbb{R}$, $\eta_{y,t+1} \sim \mathcal{N}(0,1)$, i.i.d. and ν_{t+1} is as before.

(a) Explain why this model cannot be solved in closed-form without exploiting an approximation.

(b) Relate the log-asset return of the consumption claim to the wealth-consumption ratio, exploit a Campbell-Shiller approximation, and determine the coefficients.

(c) Explain verbally how the risk-free interest rate varies in response to the long-run-risk factor y_t.

Problem 2 (Long-Run Risk and the Timing Premium)

Consider a representative agent with recursive preferences of the Epstein-Zin type as in the previous problem

$$\mathcal{U}_t = \left[(1-\beta)C_t^{1-1/\psi} + \beta\Big(\mathbb{E}_t\Big[\mathcal{U}_{t+1}^{1-\gamma}\Big]\Big)^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}}.$$

Consumption follows

$$\begin{aligned}\Delta c_{t+1} &= \mu_c + y_t + \sigma\eta_{c,t+1}\\ y_{t+1} &= \rho_y y_t + \phi_y \sigma \eta_{y,t+1}\end{aligned}$$

where $\eta_{c,t+1}, \eta_{y,t+1}$ are standard normal innovations, mutually independent and i.i.d. over time. The *timing premium* π is the maximum fraction of the consumption stream $(C_t)_{t=0}^{\infty}$ you give up in order for all risk to be resolved next period. Define the consumption stream $C_t^* = (1-\pi)C_t$ where the agent learns the realizations of $(C_t)_{t=0}^{\infty}$ with certainty at $t=1$.

Let $\mathcal{U}_0$ be the utility index of the consumption process $(C_t)_{t=0}^{\infty}$ with risk resolved gradually over time, and let $\mathcal{U}_0^*$ be the utility index of the alternative process $(C_t^*)_{t=0}^{\infty}$ where all risk is resolved at time 1. The timing premium is then defined as

$$\pi = 1 - \frac{\mathcal{U}_0}{\mathcal{U}_0^*}.$$

1. Show that for unit EIS (i.e., $\psi \to 1$), the utility index can be expressed as

$$\mathcal{U}_t = C_t^{1-\beta}\big(\mathbb{E}_t[U_{t+1}^{1-\gamma}]\big)^{\frac{\beta}{1-\gamma}}.$$

From now on, we assume $\psi = 1$.

2. Guess and verify that $\mathcal{U}_0$ is given by

$$\log(\mathcal{U}_0) = c_0 + \frac{\beta}{1-\beta\rho_y} y_0 + \frac{\beta}{1-\beta}\mu_c + \frac{1-\gamma}{2}\frac{\beta\sigma^2}{1-\beta}\left(1 + \frac{\beta^2\phi_y^2}{(1-\beta\rho_y)^2}\right)$$

3. Show that the continuation utility $\mathcal{U}_1^*$ generated by $(C_t^*)_{t=0}^{\infty}$ is given by

$$\log \mathcal{U}_1^* = c_0 + \sum_{t=1}^{\infty} \beta^{t-1} \Delta c_t$$

and use this representation to determine $\mathcal{U}_0^*$ and to derive

$$\pi = 1 - \exp\Big[\frac{1-\gamma}{2}\frac{\beta^2\sigma^2}{1-\beta^2}\Big(1 + \frac{\beta^2\phi_y^2}{(1-\beta\rho_y)^2}\Big)\Big].$$

4. Interpret how and why the preference parameters β, γ affect the timing premium.