Capital Markets and Asset Pricing
Goethe Business School
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Problem Set 4

Problem 4.1 (Portfolio Selection Theory) The expected returns of two assets A and B are 8% and 12%. The volatilities are 15% and 20%. Besides, the covariance of the returns is 1.8%.

(a) Determine the expected return and volatility of a portfolio investing 30% of wealth in asset A and 70% in asset B.

(b) Determine the portfolio weights of the minimum-variance portfolio consisting of the two assets A and B only.

(c) Determine the MVP's expected return and volatility, and interpret your results

(d) Is there a diversification effect if the returns are not correlated?

Problem 4.2 (Capital Asset Pricing Model) Suppose you have estimated a stock volatility of $\sigma_i = 22\%$. Ignoring Roll's critique you proxy the market portfolio by an index, whose volatility is $\sigma_M = 16\%$. Besides you estimate a correlation between the stock return and the market return of 54%.

(a) Determine the covariance between the asset and the market and the asset's beta factor.

(b) Suppose now the risk-free rate is 1.5% and you have observed a market risk premium of 6.5%. What is the expected market return? Depict the Security Market Line and explain its components.

(c) What is the expected return of the asset from (a) according to the CAPM?

(d) Suppose that the CAPM holds true. What would happen if the expected return were 7.5%?

(e) Determine the asset's unsystematic risk.

Problem 4.3 (Arbitrage Pricing Theory) We consider a two-factor APT and assume that idiosyncratic risk can be disregarded. There are two traded stocks and a risk-free asset.

(a) Explain how the model can be estimated and write down the security market line.

The risk-free rate is 1.5%. The factor loadings and expected returns of the two stocks are summarized in the following table.

Security	b_{i1}	b_{i2}	Exp. return
A	0.25	0.8	16.2%
B	0.75	0.9	21.6%

(b) Determine the risk premia of the two factors.

(c) You have 500 euros available. If you invest 800 euros in stock 1 and short sell 300 euros of stock 2, what are the sensitivities of your portfolio to the two factors? What is the expected return of that portfolio?

(d) Construct a portfolio consisting of assets A and B which is insensitive to the first factor.

(e) Determine the expected return of an asset with sensitivities $\beta_{i,1} = 0.8$, $\beta_{i,2} = -0.2$.

Problem 4.4 (Fama-French Model and Carbon Premium) Consider the following Fama-French-three-factor-model.

Factor	Market	Size	Book
Premium	4.8%	3.6%	5.2%

(a) Suppose there is an asset for which all factor sensitivities are positive. Characterize this asset.

(b) Consider an asset where the sensitivities are $\beta_{i,M} = 0.9$, $\beta_{i,size} = -0.5$, $\beta_{i,book} = 0.6$. Predict its expected return if the risk-free rate is 1%.

(c) Suppose now there is an asset whose sensitivities are $\beta_{i,M} = 0.8$, $\beta_{i,size} = 0.5$. The expected rate of return is 9.2%. Determine its sensitivity to the BTM factor.

Following Bolton & Kaspercyk (2021), we now consider an extension of the Fama-French model where carbon intensity is added as a factor to the model. Bolton & Kaspercyk (2021) find that there exist a *carbon premium*, i.e., more carbon-intensive industries have higher expected returns.

(d) Explain why there might be a carbon premium.

(e) Suppose we regress returns against the Fama-French factors and the logarithm of total emissions generated by the firm.

$$R_{i,t} = \alpha_i + \beta_{i,market} \cdot (R_{m,t} - r_f) + \beta_{i,size} \cdot SMB_t + \beta_{i,book} \cdot HML_t + \beta_{i,carbon} \log(E_{i,t}) + \varepsilon_{i,t},$$

where the emission factor carries a risk premium $\lambda_{carbon} = 1.5\%$. Suppose $\beta_{i,carbon} = 0.3$ (the other sensitivities might be affected by adding a new factor to the model). What does this mean in terms of expected returns, risk, and volatility?