Capital Markets and Asset Pricing
Goethe Business School
Summer Term 2022
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Problem Set 3

Problem 3.1 (State Pricing and Default Risk) Consider a firm that is financed by equity and debt. The current stock price is $S_0 = 50$. This firm has emitted two types of bonds with notional $N = 100$ each: A senior bond currently trading at 95 and a junior bond currently trading at 89. Assume that the firm faces significantly default risk. After one year, the following three scenarios can occur:

– Scenario 1: No default has occurred, and the stock price has increased by 20%.

– Scenario 2: The firm has gone bankrupt and the LGD of the junior bond is 40%.

– Scenario 3: The firm has gone bankrupt and the LGD of the senior bond is 40%, while the junior bond has been wiped out.

(a) Visualize this one-period state pricing model by a tree diagram.

(b) Determine the prices of the elementary securities and check whether the market is free of arbitrage. What would happen if the junior bond were trading at 81?

(c) Determine the risk-free rate in this economy for both discretely compounded and continuously compounded interest rates.

(d) Determine the risk-neutral survival probabilities of both bonds.

(e) Determine the price of a put option on the senior bond with a strike price of $K = 100$, and explain why it hedges the senior bond's default risk.

Problem 3.2 (Merton's Firm Value Model) Consider a firm with a firm value of 100,000,000 Euro. The asset volatility is $\sigma = 0.1$. The firm is financed by equity and a zero bond with notional 90,000,000 and maturity at $T = 10$. The risk-free rate is $r = 3.5\%$.

(a) Determine the market price of equity and debt as well as the firm's leverage.

(b) Calculate the credit spread, i.e., the yield spread between the corporate bond and the risk-free rate.

Problem 3.3 (Credit and Liquidity Premium) Suppose that a US-firm has issued two types of bonds L, I that are traded on two different markets. While both bonds are exposed to credit risk, they differ in their liquidity. The liquid bonds (L) can be traded without bid-ask spread, while the illiquid bonds (I) are facing a bid-ask-spread and a liquidity premium. The following table summarizes the data.

T	$P_{0,L}$	$P_{0,I}$	c	N
1	101.15	100	1.0%	100
2	98.00	96.7	2.0%	100

The reported prices $P_{0,L}$, $P_{0,I}$ are the current mid-prices of the liquid and illiquid bonds, respectively. The illiquid bonds are currently trading at a bid-ask-spread of 0.3%.

(a) Determine the bid price and the ask price of the illiquid bonds.

(b) Determine the liquidity premium of the illiquid bonds.

(c) Suppose that the yield curve of the corresponding US Government bonds (same notional, payment dates, and coupon rates as the corporate bonds) is given by $y(1) = -0.25\%$, $y(2) = 2.5\%$. Determine the credit spread of all bonds.