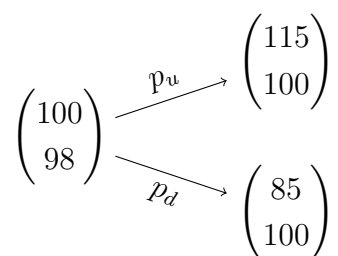


**Capital Markets and Asset Pricing**  
**Goethe Business School**  
**Summer Term 2022**  
**Dr. Christoph Hambel**  
**Problem Set 2**

**Problem 2.1 (State Pricing)** Consider the following state pricing model with two assets (stock and risk-free asset) and two states:



- (a) Calculate the prices of the elementary securities in this model.
- (b) Is this market free of arbitrage? Is the market complete? Explain your answer.
- (c) Determine the risk-free rate and the risk-neutral probabilities.
- (d) Determine the price of a European put option on the stock with a strike price of 100.
- (e) Provide the replication strategy for the put option from part (d).
- (f) Determine the expected stock return under the risk-neutral probability measure. What do you observe?

**Problem 2.2 (Option Pricing in a Two-Period Model)** Consider a two-period binomial tree with two assets (stock and risk-free asset). Today's stock price is 100. In each period, the stock price can either increase by 12% or decrease by 12%. A period corresponds to six months. The TSIR is flat and the annual interest rate is 4.04%.

- (a) Determine the spot rate over one period (= 6 months).
- (b) Set up the binomial tree for the stock price.

- (c) Calculate the risk-neutral probabilities.
- (d) Determine the price of a European call option with a strike price of 90.

**Problem 2.3 (Black-Scholes Model)** Consider a Black-Scholes model with the following parameters: the risk-free rate is  $r = 0.5\%$ , the volatility is  $\sigma = 22\%$ , and the current stock price is  $S_0 = 95$ .

- (a) Determine the price of a European call option with strike price  $K = 100$  and maturity at  $T = 2$ .
- (b) Write down the replication portfolio for the option from part (a).
- (c) Suppose you want to price a reverse convertible bond with  $N = 100$ ,  $k = 1$  paying coupons at a rate of  $c = 2\%$  and maturing in 2 years. Suppose the term structure of interest rates is flat and there is no credit risk.
- (d) Explain why the drift rate of the stock price  $\mu$  does not show up in the Black-Scholes formula.