

**Capital Markets and Asset Pricing**  
**Goethe Business School**  
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**Problem Set 1**

**Problem 1.1 (Bond Pricing)** You have a bond with notional  $N = 1,000$  which matures in 240 days (30/360 usance). The coupon is 6% and the clean price is 990.

- (a) Calculate the accrued interest and the dirty price. Explain the difference between clean and dirty bond prices.
- (b) Assume capital market usance and redo problem (a).

**Problem 1.2 (Discretely compounded vs. Continuously)** Current prices of zero-coupon bonds ( $N = 100$ ) with maturities in 6 months, and twelve months are  $P_0(T = 0.5) = 99.8$ , and  $P_0(T = 1) = 98.4$

- (a) Determine the discretely compounded spot rates  $r(0.5)$ ,  $r(1)$ , and the forward rate for an investment over the second period  $f(0.5, 1)$ .
- (b) Determine the corresponding continuously compounded interest rates.

**Problem 1.3 (TSIR and Bond Pricing)** Suppose the following bond data of German Bundesanleihen is given

	$T$	$P_0$	$c$	$N$
1st Bond	1	101.15	1.0%	100
2nd Bond	2	98.00	2.0%	100
3rd Bond	3	99.50	1.5%	100

- (a) Back out the spot and forward rates from this bond data. What kind of term structure do you get? What do you observe?
- (b) Determine the current yield, simple redemption yield, and yield-to-maturity for the second bond. Explain the differences between these yield concepts.

- (c) Suppose there is another German Bundesanleihe with a coupon rate of 3% maturing in 3 years. Determine its arbitrage-free price.
- (d) What would you do if this bond were trading at 102?
- (e) What would be different if the bonds were US Treasuries? Write down the pricing equations using spot rates.

**Problem 1.4 (Interest Rate Exposure)** We consider a coupon bond with a coupon rate of 2% per annum, a notional of 100 euros, and a time to maturity of 3 years. Coupons are paid annually. Its continuously compounded yield-to-maturity is 1%.

- (a) Determine the bond's price  $P_0$ , duration  $D$ , and convexity  $\Gamma$ .
- (b) What change in the yield-to-maturity does the duration predict if the price goes down by \$1?
- (c) Suppose you observe rising interest rate and the yield-to-maturity goes down by 80bps=0.8%. Predict the price change using (i) duration only, (ii) duration and convexity, and compare it to (iii) the true price change.
- (d) Suppose you hold an equally weighted portfolio of the bond above and a zero-coupon bond with a maturity in 5 years and a current price of 89. Determine the portfolio duration and convexity.