Agenda

Standard Lucas Tree Model

- CRRA utility and normally-distributed i.i.d. consumption growth
- Yields constant risk-fee rate, equity premium, price-dividend ratio
- Not able to explain high equity premium and low interest rate
- Main Reasons: Too smooth consumption streams, too simplified preferences
- Recursive utility helps, but is not sufficient

$$\mathrm{rp}_t^i = \frac{\theta}{\psi}\sigma_{i,c} + (1-\theta)\sigma_{i,x} \quad = \gamma \sigma_{i,c}$$

$$r_t^f = \delta + \frac{1}{\psi}\mu_c - \frac{1}{2}\frac{\theta}{\psi^2}\sigma_c^2 - \frac{1}{2}(1-\theta)\sigma_x^2.$$

Campbell & Cochrane Model

- Power utility with habit formation driven by the (unobservable) surplus-consumption ratio.
- Yields state-dependent risk aversion.
- Can explain high equity premium and low interest rate, which respond to business cycles.

$$\mathrm{rp}_t = \gamma \mathrm{cov}(r_{t+1}, \Delta c_{t+1}) + \gamma \lambda(s_t) \mathrm{cov}(r_{t+1}, \Delta c_{t+1})$$
$$r_t^f = \delta + \gamma \mu_c - \frac{1}{2}\gamma^2 \sigma_c^2 - \gamma \varphi (s_t - \bar{s}) - \frac{1}{2}\gamma^2 \left(2\lambda(s_t) + \lambda(s_t)^2\right)\sigma_c^2$$

- Model still requires unrealistically high levels of risk aversion.

Bansal & Yaron Model

- Recursive utility combined with non-i.i.d. consumption growth
- State variables: long-run risk factor and stochastic volatility
- Can explain high equity premium and low interest rate, which respond to the state variables

$= \gamma\sigma_c$

$\psi = 1.5$

$\gamma = 10$

$$\mathrm{rp}_t = \beta_c \sigma_t^2 \lambda_c + \beta_y \sigma_t^2 \lambda_y + \beta_\sigma \sigma_\sigma^2 \lambda_\sigma$$

$$r_t^f = \delta + \frac{1}{\psi}\left(\mu_c + y_t + \frac{1}{2}\sigma_t^2\right) - \frac{1}{2}\gamma\left(1 + \frac{1}{\psi}\right)\sigma_t^2$$

$$-\frac{1}{2}(1-\theta)\kappa_1^2 A_\sigma^2 \sigma_v^2 - \frac{1}{2}\left(\gamma - \frac{1}{\psi}\right)\left(1 - \frac{1}{\psi}\right)\left(\frac{\kappa_1 \psi_y}{1 - \kappa_1 \rho}\right)^2 \sigma_t^2$$

- Model generates state-dependent price-dividend ratio (Campbell-Shiller approximation)

$$z_t = A_0 + A_y y_t + A_\sigma \sigma_t^2$$

- Only works for recursive preferences with $\psi > 1$ and relatively high RRA

Barro Model

- CRRA utility and fat-tailed-distributed i.i.d. consumption growth due to disaster shocks
- Yields constant risk-fee rate, equity premium, price-dividend ratio
- Can explain high equity premium and low interest rate

$$r_t^f = \delta + \gamma\mu_c - \frac{1}{2}\gamma^2\sigma_c^2 - p\left(\mathbb{E}_t[(1-b_{t+1})^{1-\gamma}] - 1\right)$$
$$\mathrm{rp}_t^i = \gamma\sigma_c^2 + p\mathbb{E}_t\left[b_{t+1}\left((1-b_{t+1})^{1-\gamma} - 1\right)\right]$$

- Even works for CRRA utility and constant disaster size, but can be extended into several dimensions