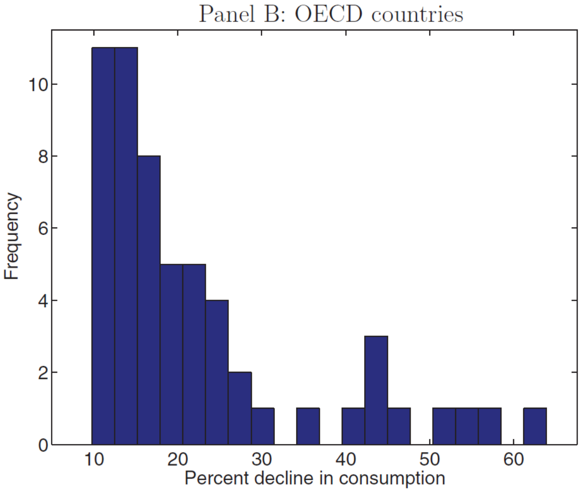
Agenda

Motivation

- In reality, consumption and stock market dynamics are not as smooth as the Lucas model suggests.
- The frameworks we have studied so far do not take into account the possibility of rare disasters that hit the economy with a slam.
- Examples:
  - Corona Crash of Spring 2020
  - Financial Crisis of 2008
  - Wall Street Crash of 1929
  - ....
- All these crashs destroyed a lot of money within some few trading days and had a significant impact on consumption.
- We study a simple model with instantaneous disaster shocks following Rietz (1998), Barro (2006, 2009).

Motivation



Selected disasters in history, see Wachter (2013), original data from Barro and Ursa (2008).

Barro (2006, 2009) – Model Setup

- Endowment economy with disaster risk component

$$\Delta c_{t+1} = \mu_c + \sigma_c \underbrace{\eta_{c,t+1}} + \nu_{t+1}$$

- $\eta_{c,t+1} \sim \mathcal{N}(0,1)$, i.i.d. and ν_{t+1} satisfies

$$\mathbb{P}\big(\underbrace{\nu_{t+1} = \log(1 - b_{t+1})}\big) = p, \qquad \mathbb{P}(\nu_{t+1} = 0) = 1 - p$$

  where $\boxed{b_{t+1}}$ is i.i.d.
- Disaster probability: p
- Loss if a disaster hits the economy: b_{t+1}

$$\begin{aligned} C_{t+1} &= C_t e^{\mu_c + \sigma_c \eta_{c,t+1} + \nu_{t+1}} \\ &= \begin{cases} C_t e^{\mu_c + \sigma_c \eta_{c,t+1}}, \; \checkmark & \text{in normal times} \\ C_t e^{\mu_c + \sigma_c \eta_{c,t+1}} (1 - \underbrace{b_{t+1}}), & \text{if a disaster hits} \end{cases} \end{aligned}$$

Calibration

- Expected consumption growth

$$\mathbb{E}_t\left[\frac{C_{t+1}}{C_t}\right] = \mu_c + \frac{1}{2}\sigma_c^2 - p\mathbb{E}_t[b_{t+1}]$$

- Rare but heavy disasters: Barro (2006, 2009) uses
  - expected growth rate without disasters $\mu_c = 2.5\%$
  - consumption volatility in normal times $\sigma_c = 2\%$
  - disaster probability $p = 1.7\%$
  - expected disaster size $\overline{b} = \mathbb{E}_t[b_{t+1}] = 29\%$
- Therefore, expected growth rate is $\mathbb{E}_t\left[\frac{C_{t+1}}{C_t}\right] \approx 2\%$.
- Alternative calibrations can be found in Wachter (2013), Barro and Jin (2011), among others.

BKY, etc
$\underline{\mu_c = 2\%}$

Agenda

General Procedure

- Consider an investor with CRRA utility (Barro 2006):

$$u(C) = \frac{1}{1-\gamma} C^{1-\gamma}.$$

- Consumption growth is i.i.d., so wealth-consumption ratio constant, and everything works along the lines of Section 2.
- Standard Procedure
  - Write down the Euler equation and determine the pricing kernel
  - Derive the risk-free interest rate.
  - Derive the equity premium.
  - Determine the wealth-consumption ratio.
- This procedure also works for recursive utility (Barro 2009).

Risk-free Rate

- Euler Condition remains unchanged in the CRRA case,

$$\mathbb{E}_t\left[\underbrace{\mathrm{e}^{-\delta}\frac{u_c(C_{t+1})}{u_c(C_t)}}_{M_{t,t+1}} R_t\right] = 1.$$

- For the risk-free asset

$$\mathrm{e}^{-r_t^f} = \mathbb{E}_t\left[\mathrm{e}^{-\delta}\frac{u_c(C_{t+1})}{u_c(C_t)}\right]$$

- Exploiting a first order approximation ($\mathrm{e}^{-p} \approx 1 - p$):

$$r_t^f = \delta + \gamma\mu_c - \frac{1}{2}\gamma^2\sigma_c^2 - p\left(\mathbb{E}_t[\underbrace{(1 - b_{t+1})}_{\text{heavily depends on } \gamma}{}^{1-\gamma}] - 1\right)$$

heavily depends on γ

  - blue terms: standard
  - red terms: disaster risk reduces the interest rate

Equity Premium

- Equity premium

$$\mathrm{rp}_t^i = \gamma\sigma_c^2 + p\mathbb{E}_t\left[b_{t+1}\left((1-b_{t+1})^{1-\gamma}-1\right)\right]$$

  - blue terms: standard
  - red terms: disaster risk boosts the equity premium

- The benchmark calibration generates a high equity premium and a low real interest rate even with CRRA utility and a moderate degree of risk aversion in the range between $\boxed{\gamma = 3 \text{ and } \gamma = 4.}$

B&Y rely on $\gamma = 10$

Possible Model Extensions

Barro & B&Y

EZ

- The model is still very simplistic.
  - equity premium is constant.
  - interest rates is constant.
  - financial crises are point events.
  - CRRA implies that an increase in uncertainty (σ, p, or b_{t+1}) for given expected consumption growth implies a higher price-dividend ratio.
- It can be adjusted in various dimension to account for
  - recursive preferences (e.g., Barro 2009; Barro and Jin 2011)
  - long-run risk and stochastic volatility (as in Bansal and Yaron 2004)
  - time-varying disaster risk (e.g., Wachter 2013)
  - periods of crisis (e.g., Branger et al. 2016)
  - climate-induced disaster shocks (e.g., Hambel et al. 2020)
  - ...

Possible Model Extensions

Wachter 2013, JF

$$\Delta p_{t+1} = \theta(p_t - \bar{p}) + \sigma_p \sqrt{p_t}\, \underbrace{\eta^p_{t+1}}$$

$$\xrightarrow{\Delta t \to 0} \quad d\lambda_t = \theta(\lambda_t - \bar{\lambda}) + \sigma_\lambda \sqrt{\lambda_t}\, dW_t$$

$$\lambda_t > 0 \text{ a.s.}$$

Branger, Kraft, Meinerding, RFS, 2016

Markov chain with 2 states

$$\boxed{b} \underset{\lambda^b}{\overset{\lambda^r}{\rightleftarrows}} \boxed{r}$$

$$\Delta c_{t+1} = \mu_b + \sigma_b \eta_{t+1} + \nu^r_{t+1} \quad \text{if } b$$

$$\Delta c_{t+1} = \mu_r + \sigma_r \eta_{t+1} + \nu^b_{t+1} \quad \text{if } r$$

T_t, $\quad \lambda_t = \lambda_0 + \lambda_1 T_t$

Hambel et al 2020 WP

Vardas & Xepapadeas (2019)

$$d\lambda_t = \theta\left(\lambda_t - (\lambda_0 + \lambda_1 T)\right)dt + \sigma\sqrt{\lambda_t}\,dW$$