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1. Option Pricing in Partial Equilibrium
2. General Equilibrium Asset Pricing
3. Habit Formation and Asset Pricing
4. Recursive Utility
5. Long-Run Risk and Asset Pricing
   - Model Setup
   - Indirect Utility and Wealth-Consumption Ratio
   - Pricing Kernel, Risk-Free Rate, and MPR
   - Pricing of the Dividend Claim

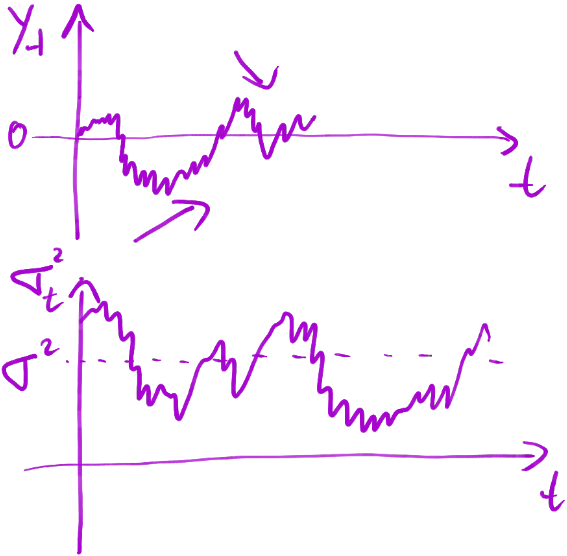
Bansal and Yaaron (2004) – Model Setup

- Endowment economy with long-run risk component and stochastic volatility

$$
\begin{aligned}
\Delta c_{t+1} &= \mu_c + y_t + \sigma_t \eta_{c,t+1} \quad \phi_d, \psi_d > 1 \\
\Delta d_{t+1} &= \mu_d + \phi_d y_t + \psi_d \sigma_t \left(\rho_{cd} \eta_{c,t+1} + \sqrt{1 - \rho_{cd}^2} \eta_{d,t+1} \right) \\
\Delta y_{t+1} &= (\rho - 1) y_t + \psi_y \sigma_t \eta_{y,t+1} \\
\Delta \sigma_{t+1}^2 &= (\nu - 1)(\sigma_t^2 - \sigma^2) + \sigma_v \eta_{v,t+1}
\end{aligned}
$$

- Recursive Preferences with risk aversion γ and EIS ψ.
- In Bansal and Yaaron (2004), the notation is slightly different, LRR-factor is denoted by x.
- Remarkable properties:
  - Stochastic volatility σ.
  - Long-run risk component in y in consumption and dividend dynamics.
  - Dividends are potentially more volatile than consumption.

Bansal and Yaaron (2004) – The long-run risk factor



Bansal and Yaron (2004) – Solution Approaches

- No closed-form solution available as returns are not normally distributed.
- Numerical solution approach
  - Done by Bansal and Yaron (2004).
  - Numerical solutions are hard to interpret.
  - Requires a lot of analyses to point out how a certain parameter affects the solution.
- Approximate closed-form solution can be achieved
  - Done by Bansal and Yaron (2004) to gain intuition.
  - Approximate growth rates by a linear function of state variables, see Campbell-Shiller (1998).
  - Changes in growth rates approximately follow a joint normal distribution.

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Campbell-Shiller Approximation

- The Campbell-Shiller (1988)-approximation linearizes the relation between asset returns, dividend growth and price dividend-ratios.
- Log-return of an asset

$$z_t = \log \frac{P_t}{D_t}$$

$$+\boxed{\log(D_{t+1})} - \log(D_{t+1})$$

$$
\begin{aligned}
r_{t+1} &= \log\left(\frac{P_{t+1} + D_{t+1}}{P_t}\right) \\
&= \log(P_{t+1} + D_{t+1}) - \log(P_t) \\
&= \log\left(\frac{P_{t+1} + D_{t+1}}{D_{t+1}}\right) + \log(D_{t+1}) - \log(P_t) \\
&= \log\left(1 + \frac{P_{t+1}}{D_{t+1}}\right) + \log(D_{t+1}) - \log(D_t) + \log(D_t) - \log(P_t) \\
&= \boxed{\log\left(1 + e^{z_{t+1}}\right)} + \Delta d_{t+1} - z_t
\end{aligned}
$$

where $z_{t+1} = \log(P_{t+1}/D_{t+1})$ is the log price-dividend ratio.

Campbell-Shiller Approximation

- First-order Taylor approximation to the function $f(z) = \log(1 + e^z)$ around the average log price-dividend ratio $\overline{z} = \overline{p} - \overline{d}$.

$\overline{z} \approx 20$

Campbell-Shiller Approximation

The log return r_{t+1} of an asset with dividend growth rate Δd_{t+1} and log price-dividend ratio z_t is approximately equal to

$$\boxed{r_{t+1} \approx \kappa_0 + \underbrace{\kappa_1 z_{t+1} - z_t}_{\approx z_{t+1} - z_t = \Delta z_{t+1}} + \Delta d_{t+1}}$$

with

$$\kappa_0 = \log(1 + e^{\overline{z}}) - \overline{z}\frac{e^{\overline{z}}}{1 + e^{\overline{z}}}, \qquad \kappa_1 = \underbrace{\frac{e^{\overline{z}}}{1 + e^{\overline{z}}}}_{\approx 1} < 1.$$

Proof. In class □

- Remark: For unit EIS, the Campbell-Shiller approximation is the correct solution.

Proof: Campbell-Shiller Approximation

$$r_{t+1} = \log\left(1 + e^{z_{t+1}}\right) + \Delta d_{t+1} - z_t$$

Taylor-expansion

$z_0 = \bar{z}$

avg log PD-ratio

$$f(z) = \log(1 + e^z)$$

$$f'(z) = \frac{e^z}{1 + e^z}$$

$$f(z) \approx f(z_0) + f'(z_0)[z - z_0]$$

$$r_{t+1} \approx f(\bar{z}) + f'(\bar{z})[z_{t+1} - \bar{z}] + \Delta d_{t+1} - z_t$$

$$= \log(1 + e^{\bar{z}}) + \frac{e^{\bar{z}}}{1 + e^{\bar{z}}}[z_{t+1} - \bar{z}] + \Delta d_{t+1} - z_t$$

Proof: Campbell-Shiller Approximation

$$= \underbrace{c_g\left(1+e^{\bar z}\right) + \bar z \frac{e^{\bar z}}{1+e^{\bar z}}}_{\kappa_0}$$

$$- \kappa_1 z_{t+1} + \Delta d_{t+1} - z_t$$

$$= \kappa_0 - \kappa_1 z_{t+1} + \Delta d_{t+1} - z_t \approx r_{t+1}$$

Wealth-Consumption Ratio

- Recall from the previous section: the log pricing kernel is

$$m_{t,t+1} = -\delta\theta - \frac{\theta}{\psi} g_{c,t+1} + (\theta - 1) r_{x,t+1}$$

where $\theta = \frac{1-\gamma}{1-1/\psi}$, $g_{c,t+1} = \Delta c_{t+1}$ is log-consumption growth, and $r_{x,t+1} = \Delta x_{t+1}$ is the gross return on total wealth.

- Pricing the consumption claim

$$\begin{aligned}
X_t &= \mathbb{E}_t[M_{t,t+1} X_{t+1}] \\
\iff \qquad 1 &= \mathbb{E}_t[\mathrm{e}^{m_{t,t+1} + \Delta x_{t+1}}] \\
&= \mathbb{E}_t\left[\mathrm{e}^{-\delta\theta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta-1)\Delta x_{t+1} + \Delta x_{t+1}}\right] \\
&= \mathbb{E}_t\left[\mathrm{e}^{-\delta\theta - \frac{\theta}{\psi}\Delta c_{t+1} + \theta\Delta x_{t+1}}\right] \\
&\approx \mathbb{E}_t\left[\mathrm{e}^{-\delta\theta - \frac{\theta}{\psi}\Delta c_{t+1} + \theta(\kappa_0 + \kappa_1 z_{t+1} - z_t + \Delta c_{t+1})}\right] \\
&= \mathbb{E}_t\left[\mathrm{e}^{-\delta\theta + (1-\gamma)\Delta c_{t+1} + \theta(\kappa_0 + \kappa_1 z_{t+1} - z_t)}\right]
\end{aligned}$$

Wealth-Consumption Ratio

Proposition – Affine Wealth-Consumption Ratio

Suppose that the Campbell-Shiller Approximation holds true. The wealth consumption ratio is affine in the state variables

$$\boxed{z_t = A_0 + A_y y_t + A_\sigma \sigma_t^2}$$

where

$$
\begin{aligned}
A_y &= \frac{1 - 1/\psi}{1 - \kappa_1 \rho} \\
A_\sigma &= \frac{(1-\gamma)(1-1/\psi)}{2(1-\kappa_1 \nu)} \left(1 + \left[\frac{\kappa_1 \psi_y}{1 - \kappa_1 \rho}\right]^2\right) \\
A_0 &= \dots
\end{aligned}
$$

Proof. Exercise □

Wealth-Consumption Ratio – Approach

- CS-approximation implies

$$1 \approx \mathbb{E}_t\left[\mathrm{e}^{-\delta\theta+(1-\gamma)\boxed{\Delta c_{t+1}}+\theta(\kappa_0+\kappa_1\boxed{z_{t+1}}-z_t)}\right]$$

- Substitute the conjecture $z_t = A_0 + A_y y_t + A_\sigma \sigma_t^2$ into the pricing equation.
- Simplify as much as you can and calculate the cond. expectation.
- You'll get an equation $T_0 + T_y y_t + T_\sigma \sigma_t^2 = 0$.
- This leads to a system $T_0 = 0,\ T_y = 0,\ T_\sigma = 0$.
- Solve this system for A_0, A_y, and A_σ.

Discussion of Wealth-Consumption Ratio

- Exposure to long-run risk y_t

$$z_t = A_0 + A_y y_t + A_\sigma \sigma_t^2$$

$$A_y = \frac{1 - 1/\psi}{1 - \underbrace{\kappa_1 \rho}_{\approx 1\ <1}}$$

$$e^{\bar{z}} \approx 25$$

  - From the data, we know that $P/D = \mathrm{e}^z \approx 25$.
  - Therefore, $\kappa_1 = \frac{\mathrm{e}^{\bar{z}}}{1+\mathrm{e}^{\bar{z}}} \approx 1$.
  - Since $\rho < 1$, the denominator is positive.
  - Exposure to LRR is positive iff $\boxed{\psi > 1.}$

- Exposure to stochastic volatility

$$A_\sigma = \frac{(1-\gamma)(1-1/\psi)}{2(1-\kappa_1\nu)} \underbrace{\left(1 + \left[\frac{\kappa_1 \psi_y}{1-\kappa_1\rho}\right]^2\right)}_{>0}$$

$$\Big| : \left(1 - \frac{1}{\psi}\right)^2$$

  - Since $\nu < 1$, the denominator is positive.
  - Exposure to stochastic volatility is positive iff $(1-\gamma)(1-1/\psi) > 0$, i.e., $\theta > 0$. $\Rightarrow \theta < 1 \iff \theta = \frac{1-\gamma}{1-1/\psi} > 0$

Indirect Utility

$z_t = \log\left(\frac{X_t}{C_t}\right)$

- The indirect utility is given by

$$J_t = \alpha^{\frac{1}{1-1/\psi}} \left(\frac{C_t}{X_t}\right)^{\frac{1}{1-\psi}} X_t$$
$$= \alpha^{\frac{1}{1-1/\psi}} e^{z_t \frac{1}{\psi-1}} X_t$$

  - $\psi > 1$: indirect utility is increasing in wealth-consumption ratio.
  - investor does not care much about consumption smoothing over time, substitution effect dominates wealth effect.
  - The opposite is true for $\psi < 1$.
- How does J react to variation in the state variables?

Indirect Utility and LRR

- The indirect utility is approximately given by

$$J_t \approx \alpha^{\frac{1}{1-1/\psi}} \mathrm{e}^{\overbrace{(A_0 + A_y y_t + A_\sigma \sigma_t^2)}^{z_t} \frac{1}{\psi - 1}} X_t$$

- Influence of the LRR-factor

$$\frac{\partial J_t}{\partial y_t} \approx \frac{\overset{>0}{\alpha^{\frac{1}{1-1/\psi}}}}{\underbrace{(1 - \kappa_1 \rho)\psi}_{>0}} \underbrace{\mathrm{e}^{(A_0 + A_y y_t + A_\sigma \sigma_t^2) \frac{1}{\psi - 1}} X_t}_{>0} > 0$$

- High y is always good news. For larger y, investment opportunities become more attractive.
- Consider the case $\psi > 1$
  - Agent wants to smooth less over time than the log-investor.
  - Substitution effect dominates the income effect.
  - She reacts to good investment opportunities by saving more and consuming less, which increases his wealth-consumption ratio.
  - Wealth increases as consumption is exogenous.

Indirect Utility and Stochastic Volatility

- Influence of the sochastic volatility

$$\frac{\partial J_t}{\partial \sigma_t} \approx \underbrace{\frac{(1-\gamma)\overbrace{\alpha^{\frac{1}{1-1/\psi}}}^{>0}}{(1-\kappa_1\rho)\psi}}_{>0} \underbrace{\mathrm{e}^{z_t \frac{1}{\psi-1}} X_t}_{>0} \underbrace{\frac{1}{2(1-\kappa_1\nu)}}_{>0} \underbrace{\left(1+\left[\frac{\kappa_1\psi_y}{1-\kappa_1\rho}\right]^2\right)}_{>0}$$

- High volatility is thus
  - bad news for $\boxed{\gamma > 1}$: Investor worries about increased uncertainty. $\frac{\partial J_t}{\partial \sigma_t} < 0$
  - good news for $\gamma < 1$: investor is happy about upside potential. $\frac{\partial J_t}{\partial \sigma_t} > 0$
- Consider the case $\boxed{\psi > 1,\ \gamma > 1}$:
  - large σ_t is bad news for the investor.
  - investor with $\psi > 1$ reacts to bad investment opportunities by consuming more today.
  - wealth-consumption ratio decreases $(\underbrace{A_\sigma < 0}_{\Leftrightarrow\, \psi > 1})$.

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Plan for Asset Pricing

- We can calculate the pricing kernel using the Campbell-Shiller approximation
- The pricing kernel dynamics will give further insight on
  - risk-free rate
  - market prices of risk
- Once we have the pricing kernel, we can calculate the price of the dividend claim and its risk premium.

Pricing Kernel

- Remember, the log pricing kernel is given by

$$m_{t,t+1} = -\delta\theta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1)r^x_{t+1}$$

- Substitute the Campbell-Shiller approximation into the log pricing kernel

$$m_{t,t+1} = -\delta\theta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1)(\kappa_0 + \kappa_1 z_{t+1} - z_t + \Delta c_{t+1})$$

- It is easy to check that $\theta - 1 - \frac{\theta}{\psi} = -\gamma$, hence

$$m_{t,t+1} = -\delta\theta - (1-\theta)\kappa_0 + (1-\theta)z_t - (1-\theta)\kappa_1 z_{t+1} - \gamma\Delta c_{t+1}$$

- Substitute the solution for the wealth-consumption ratio into the log pricing kernel

$$
\begin{aligned}
m_{t,t+1} = &-\delta\theta - (1-\theta)\kappa_0 + (1-\theta)\overbrace{(A_0 + A_y y_t + A_\sigma \sigma_t^2)}^{z_t} \\
&- (1-\theta)\kappa_1 \underbrace{(A_0 + A_y y_{t+1} + A_\sigma \sigma_{t+1}^2)}_{z_{t+1}} - \Delta c_{t+1}
\end{aligned}
$$

- Substitute the dynamics of the state variables into the log pricing kernel. We end up with

$$
\begin{aligned}
m_{t,t+1} = &-\delta\theta - (1-\theta)\kappa_0 + (1-\theta)(1-\kappa_1)A_0 - \gamma\mu_c \\
&- (1-\theta)\kappa_1 A_\sigma (1-\nu)\sigma^2 \\
&+ \left[(1-\theta)A_y(1-\kappa_1\rho) - \gamma\right] y_t \\
&+ (1-\theta)A_\sigma(1-\kappa_1\nu)\sigma_t^2 \\
&- (1-\theta)\kappa_1 A_y \psi_y \sigma_t \eta_{y,t+1} - (1-\theta)\kappa_1 A_\sigma \sigma_v \eta_{v,t+1} \\
&- \gamma\sigma_t\eta_{c,t+1}.
\end{aligned}
$$

$\theta < 1$

$\theta = 1$ CRRA

Pricing Kernel

$z_t = A_0 + A_y y + A_\sigma \sigma^2$

- Plugging A_0 into the pricing kernel and some painful calculations lead to

$r_c + y_t = \mathbb{E}_t[\Delta c_{t+1}]$

$$
\begin{aligned}
m_{t,t+1} = & -\delta - \frac{1}{\psi}\mu_c - \frac{1}{\psi}y_t + \frac{1}{2}\theta(1-\theta)\kappa_1^2 A_\sigma^2 \sigma_v^2 \\
& + \frac{1}{2}\left(\gamma - \frac{1}{\psi}\right)(1-\gamma)\left[1 + \left(\frac{\kappa_1 \psi_y}{1-\kappa_1\rho}\right)^2\right]\sigma_t^2 \\
& - (1-\theta)\kappa_1 A_y \psi_y \sigma_t \eta_{y,t+1} - (1-\theta)\kappa_1 A_\sigma \sigma_v \eta_{v,t+1} \\
& - \gamma\sigma_t\eta_{c,t+1}.
\end{aligned}
$$

- With EZ-utility, shocks to state variables (y_t and σ_t) are priced.
- Notice that for CRRA-utility $\theta = 1$, i.e.,

$$m_{t,t+1} = -\delta - \frac{1}{\psi}(\mu_c + y_t) - \gamma\sigma_t\eta_{c,t+1}.$$

Market Prices of Risk

Market Price of Risk

Suppose the Campbell-Shiller approximation holds true.

- The market price of risk for a shock to consumption growth $\sigma_t \eta_{c,t+1}$ is given by

$$-\frac{\partial m_{t,t+1}}{\partial(\sigma_t \cdot \eta_{t+1})} = \lambda_c = \gamma.$$

- The market price of risk for a shock to the LRR factor $\sigma_t \eta_{y,t+1}$ is given by

$$\lambda_y = (1-\theta)\kappa_1 A_y \psi_y = \left(\gamma - \frac{1}{\psi}\right)\frac{\kappa_1 \psi_y}{1-\kappa_1\rho}$$

- The market price of risk for a shock to volatility $\sigma_v \eta_{v,t+1}$ is given by

$$\lambda_\sigma = (1-\theta)\kappa_1 A_\sigma = (1-\gamma)\left(\gamma - \frac{1}{\psi}\right)\frac{\kappa_1}{2(1-\kappa_1\nu)}\left[1+\left(\frac{\kappa_1\psi_y}{1-\kappa_1\rho}\right)^2\right]$$

Some Remarks

- Bansal and Yaron (2004) define the MPR λ_c of the shock $\sigma_t \eta_{t+1}$
  $\rightarrow$ Sensitivity of the log pricing kernel w.r.t. $\sigma_t \eta_{t+1}$.
- Other authors define the MPR $\widehat{\lambda}_c$ as the sensitivity w.r.t. $\eta_{t+1} \sim \mathcal{N}(0,1)$.
  $\rightarrow$ Typically done in continuous time
- Then, $\widehat{\lambda}_c$ is the Sharpe ratio

$$\widehat{\lambda}_c = \frac{\mathrm{rp}_t}{\sigma_t} = \gamma \sigma_t$$

instead of

$$\lambda_c = \frac{\mathrm{rp}_t}{\sigma_t^2} = \gamma$$

- In continuous time, there is a crucial relation between the MPR $\widehat{\lambda}_c$ and the change of measure from $\mathbb{P}$ to $\mathbb{Q}$.

Some Remarks

- Higher market prices of risk indicate higher risk premia.
- An agent with CRRA-utility ($\theta = 1$) does not price the state variable risk:

$$\lambda_\sigma = \lambda_y = 0.$$

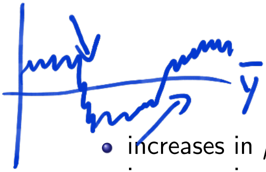
  and the market price of consumption risk is the same as for recursive utility,

$$\lambda_c = \gamma.$$

- In a model without stochastic volatility and LRR, the market price of risk for a shock to consumption growth is the same.
- Consequently, Epstein-Zin only thus does not help to solve the equity premium puzzle.

Market Price of Risk for LRR

ρ : persistence of shocks $\qquad$ ψ_y volatility of LRR risk



- Market price of risk for a shock to the LRR factor

$$\lambda_y = \underbrace{\left(\gamma - \frac{1}{\psi}\right)}_{>0} \frac{\kappa_1 \psi_y}{1 - \kappa_1 \rho}.$$

$> 0 \iff \gamma > \frac{1}{\psi} \iff \theta < 1$

  - increases in ρ, i.e., in the permanence of shocks.
  - increases in ψ_y, i.e., in volatility of shocks.
  - is positive iff $\gamma > 1/\psi$ (preferences for early resolution of uncertainty).
  - an asset which has a high payoff when investment opportunities are good makes future consumption more risky, investor would prefer to eliminate this risk today

- Market price of risk for a shock volatility

$$\lambda_\sigma = \underbrace{(1-\gamma)}_{<0}\underbrace{\left(\gamma - \frac{1}{\psi}\right)}_{>0}\underbrace{\frac{\kappa_1}{2(1-\kappa_1\nu)}}_{>0}\underbrace{\left[1 + \left(\frac{\kappa_1\psi_y}{1-\kappa_1\rho}\right)^2\right]}_{>0} \quad <0$$

  - is negative if $\boxed{\gamma > 1 \text{ and } \gamma > \frac{1}{\psi},}$ i.e., if
    - investor is more risk-averse than log investor (in this case, a high volatility is bad news and signals worse investment opportunities)
    - investor has preference for early resolution of uncertainty

    or if $\gamma < 1$ and $\gamma < \frac{1}{\psi}$, i.e., if
    - investor is less risk-averse than log investor
    - investor has preference for late resolution of uncertainty
  - increases in permanence of long-run risk shocks (ρ) and volatility shocks (ν).

Risk-Free Rate

- Pricing Equation for the risk-free asset

$$\mathbb{E}_t[\mathrm{e}^{m_{t,t+1}}] = \mathrm{e}^{-r_t^f}$$

- Therefore,

$$\begin{aligned}
r_t^f &= -\ln\left(\mathbb{E}_t[\mathrm{e}^{m_{t,t+1}}]\right) \\
&= -\mathbb{E}_t[m_{t,t+1}] \overbrace{- \frac{1}{2}\mathrm{var}_t[m_{t,t+1}]}^{\text{Jensen}} \\
&= \delta + \frac{1}{\psi}\mu_c + \frac{1}{\psi}y_t - \frac{1}{2}\theta(1-\theta)\kappa_1^2 A_\sigma^2 \sigma_v^2 \\
&\quad - \frac{1}{2}\left(\gamma - \frac{1}{\psi}\right)(1-\gamma)\left[1 + \left(\frac{\kappa_1 \psi_y}{1-\kappa_1\rho}\right)^2\right]\sigma_t^2 \\
&\quad - 0.5\lambda_c^2\sigma_t^2 - 0.5\lambda_y^2\sigma_t^2 - 0.5\lambda_\sigma^2\sigma_v^2
\end{aligned}$$

Risk-Free Rate

- Substituting the market prices of risks into the risk-free rate and some algebra leads to the following result:

Risk-free Rate

Suppose the Campbell-Shiller approximation holds true. The risk-free rate is (standard for EZ, standard but state-dependent; new components)

$$r_t^f = \delta + \frac{1}{\psi}\left(\mu_c + y_t + \frac{1}{2}\sigma_t^2\right) - \frac{1}{2}\gamma\left(1 + \frac{1}{\psi}\right)\sigma_t^2$$
$$\left\{\begin{array}{l} -\frac{1}{2}(1-\theta)\kappa_1^2 A_\sigma^2 \sigma_v^2 \\ -\frac{1}{2}\left(\gamma - \frac{1}{\psi}\right)\left(1 - \frac{1}{\psi}\right)\left(\frac{\kappa_1 \psi_y}{1-\kappa_1\rho}\right)^2 \sigma_t^2 \end{array}\right.$$

Risk-Free Rate

$\gamma = 10$

$\psi = 1.5$

- Consider the case $\psi > 1$, $\gamma > 1$, i.e., $\theta < 0$ as in Bansal and Yaron (2004).
- First three terms: standard
  - sensitivity of interest rate to consumption growth: $\frac{1}{\psi}$
  - typically much lower than γ.
  - interest rate goes down compared to CRRA.
- Impact of volatility risk depends on fourth and fifth term
  - Additional precautionary savings term for stochastic volatility.
  - sensitivity is proportional to $\left(\gamma - \frac{1}{\psi}\right)\left(1 - \frac{1}{\psi}\right) > 0$.
  - interest rate goes down compared to CRRA.

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Return on the Dividend Claim

- Consider a general dividend claim with dividend growth $\phi_d = 3$

$$\Delta d_{t+1} = \mu_d + \phi_d y_t + \psi_d \sigma_t \left(\rho_{cd} \eta_{c,t+1} + \sqrt{1 - \rho_{cd}^2} \boxed{\eta_{d,t+1}} \right)$$

$\psi_d >> 1$

- Campbell-Shiller approximation

$$r_{t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + \Delta d_{t+1}$$

- Conjecture: affine structure of the log price-dividend ratio

$$z_t = A_0 + A_y y_t + A_\sigma \sigma_t^2$$

- Return on this claim is thus

$$\begin{aligned} r_{t+1} = \kappa_0 &+ \kappa_1 (A_0 + A_y y_{t+1} + A_\sigma \sigma_{t+1}^2) \\ &- (A_0 + A_y y_t + A_\sigma \sigma_t^2) + \Delta d_{t+1} \end{aligned}$$

- Substituting everything we know into the previous equation and simplifying a lot yields...

Return on the Dividend Claim

Proposition

Suppose that the Campbell-Shiller approximation holds. With the risk factors defined as above, the return on the dividend claim is

$$\begin{aligned} r_{t+1} = & \boxed{\mu_d + \kappa_0 + (\kappa_1 - 1)A_0 + \kappa_1 A_\sigma (1-\nu)\sigma^2 \\ & + [\phi_d - (1-\kappa_1\rho)A_y]y_t - (1-\kappa_1\nu)A_\sigma\sigma_t^2} \\ & \boxed{+ \beta_y \sigma_t \underbrace{\eta_{y,t+1}} + \beta_\sigma \sigma_\sigma \underbrace{\eta_{\sigma,t+1}} \\ & + \beta_c \sigma_t \underbrace{\eta_{c,t+1}} + \beta_d \sigma_t \eta_{d,t+1}} \end{aligned}$$

$E_t[r_{t+1}]$

$\eta_{d,t+1}$ is not priced because it is idiosyncratic risk

where the risk exposures are

$\eta_y, \eta_c, \eta_\sigma$ are systematic risk, which are priced by the rh.

$$\beta_c = \rho_{cd}\psi_d, \qquad \beta_d = \sqrt{1-\rho_{cd}^2}\,\psi_d,$$

$$\beta_y = \kappa_1 A_y \psi_y, \qquad \beta_\sigma = \kappa_1 A_\sigma,$$

Return on the Dividend Claim

Proposition (continued)

... where the log price-dividend ratio is given by

$$z_t = A_0 + A_y y_t + A_\sigma \sigma_t^2$$

with

$$
\begin{aligned}
A_y &= \frac{\phi_d - \frac{1}{\psi}}{1 - \kappa_1 \rho} \\
A_\sigma &= \frac{\frac{1}{2}(\beta_y - \lambda_y)^2 + \frac{1}{2}\psi_d^2 - \rho_{cd}\psi_d\gamma + (1-\theta)(1-\kappa_1\nu)A_\sigma^{wcr} + \frac{1}{2}\gamma^2}{1 - \kappa_1 \nu} \\
A_0 &= \dots
\end{aligned}
$$

Return on the Dividend Claim

- Therefore, for the return on the dividend claim it holds

$$
\begin{aligned}
r_{t+1} = \mathbb{E}_t[r_{t+1}] \\
&+ \beta_y \sigma_t \eta_{y,t+1} + \beta_\sigma \sigma_\sigma \eta_{\sigma,t+1} \\
&+ \beta_c \sigma_t \eta_{c,t+1} + \beta_d \sigma_t \eta_{d,t+1}
\end{aligned}
$$

The betas give the risk exposures and the expected excess return is (standard, new components)

$$
\begin{aligned}
rp_t^d = \mathbb{E}_t[r_{t+1}] + \frac{1}{2}\mathrm{var}_t[r_{t+1}] - r_t^f \\
&= \beta_c \sigma_t^2 \lambda_c + \beta_y \sigma_t^2 \lambda_y + \beta_\sigma \sigma_\sigma^2 \lambda_\sigma .
\end{aligned}
$$

- Notice that the market price of dividend risk is zero.

Components of the Risk Premium

$$rp_t = \underbrace{\underbrace{\beta_c \lambda_c \sigma_c^2}_{\text{excess return on the market portfolio}}}_{I} + \underbrace{\beta_y \lambda_y \sigma_t^2}_{II} + \underbrace{\beta_\sigma \lambda_\sigma \sigma_v^2}_{III}$$

I: standard: $\beta_c = \rho_{cd} \varphi_d$, $\lambda_c = \gamma$

$$I = \rho_{cd} \varphi_d \gamma \sigma_t^2 = \gamma \underbrace{\rho_{cd} (\varphi \sigma_t) \cdot \sigma_t}$$

$$\gamma \operatorname{cov}_t(\Delta d_{t+1}, \Delta c_{t+1})$$

II: $\beta_y = \underbrace{\kappa_1 A_y \varphi_y}_{>0}$, $\lambda_y = (1-\theta) \underbrace{\kappa_1 A_y \varphi_y}_{>0} > 0$ iff $\theta < 1$

III: $\beta_\sigma = \kappa_1 \underbrace{A_\sigma}_{<0}$, $\lambda_\sigma = (1-\theta) \kappa_1 \underbrace{A_\sigma}_{<0}$ iff $\theta < 1$

wcr

Bansal and Yaron (2004): Empirical Results

γ	ψ	$E(R_m - R_f)$	$E(R_f)$	$\sigma(R_m)$	$\sigma(R_f)$	$\sigma(p-d)$
			Panel A: $\phi = 3.0$, $\rho = 0.979$			
7.5	0.5	0.55	4.80	13.11	1.17	0.07
7.5	1.5	2.71	1.61	16.21	0.39	0.16
10.0	0.5	1.19	4.89	13.11	1.17	0.07
10.0	1.5	4.20	1.34	16.21	0.39	0.16
			Panel B: $\phi = 3.5$, $\rho = 0.979$			
7.5	0.5	1.11	4.80	14.17	1.17	0.10
7.5	1.5	3.29	1.61	18.23	0.39	0.19
10.0	0.5	2.07	4.89	14.17	1.17	0.10
10.0	1.5	5.10	1.34	18.23	0.39	0.19
			Panel C: $\phi = 3.0$, $\rho = \varphi_e = 0$			
7.5	0.5	-0.74	4.02	12.15	0.00	0.00
7.5	1.5	-0.74	1.93	12.15	0.00	0.00
10.0	0.5	-0.74	3.75	12.15	0.00	0.00
10.0	1.5	-0.74	1.78	12.15	0.00	0.00

Bansal and Yaron (2004): Empirical Results

Variable	Data		Model	
	Estimate	SE	$\gamma = 7.5$	$\gamma = 10$
		Returns		
$E(r_m - r_f)$	6.33	(2.15)	4.01	6.84
$E(r_f)$	0.86	(0.42)	1.44	0.93
$\sigma(r_m)$	19.42	(3.07)	17.81	18.65
$\sigma(r_f)$	0.97	(0.28)	0.44	0.57
		Price Dividend		
$E(\exp(p - d))$	26.56	(2.53)	25.02	19.98
$\sigma(p - d)$	0.29	(0.04)	0.18	0.21