# Agenda

# Motivation for Habit Formation

- The standard Lucas tree model with CRRA utility is not able to match the relevant asset pricing moments.
- More realistic preference structure involving the agent's habit level of consumption.
- Investor does not look at consumption only, but compares it to some benchmark.
- Variation in the habit level over time
  - can be used to model business cycles.
  - leads to state dependent risk aversion.
  - can lead to time-dependent variations, e.g., counter-cyclical Sharpe ratios.

# Preferences

- Popular choices with instantaneous utility function

$$u(C,H) = \frac{(C-H)^{1-\gamma}}{1-\gamma}, \qquad u(C,H) = \frac{(C/H)^{1-\gamma}}{1-\gamma}.$$

- $H$ denotes the habit level.
- Investor compares own consumption to some benchmark which can
  1. either depend on own past consumption (internal habit)
  2. or be some exogenous process (external habit)
- An admissible consumption strategy satisfies $C_t > H_t$ for all $t = 1, \ldots, T$.

- Degree of relative risk aversion for additive habit

$$RRA(C,H) = -C\frac{u_{cc}(C,H)}{u_c(C,H)} = \gamma\frac{C}{C-H}.$$

- Risk aversion is higher in bad times, i.e., if $C - H$ is small.
- Utility index

$$\sum_{u=0}^{T} \mathbb{E}[u(C_t,H_t)]\,e^{-\delta t}$$

$$\mathcal{U} = \cancel{\sum_{k=0}^{T} u(C_t, H_t)e^{-\delta t}}$$

# Internal vs. External Habit Formation

- **Internal Habit**
  - Agent compares consumption today to average consumption in the past, e.g., Abel (1999)
  - Habit level is moving average of past consumption

$$H_t = H_0 \mathrm{e}^{-\beta t} + \alpha \sum_{s=1}^{t-1} \mathrm{e}^{-\beta(t-s)} C_s$$

  - Habit level dynamics

$$H_t = \mathrm{e}^{-\beta} H_{t-1} + \alpha \mathrm{e}^{-\beta} C_{t-1}$$

- **External Habit** is determined by consumption of other investors, e.g., Campbell and Cochrane (1999), Abel (1990).

# Agenda

# Campbell and Cochrane (1999) – Model Setup

- Endowment economy with aggregate consumption dynamics

$$\Delta c_{t+1} = \mu_c + \nu_{t+1}$$

  where the innovations $\nu_{t+1} \sim_{i.i.d.} \mathcal{N}(0, \sigma_c^2)$

- Agent has **external** habit formation with habit level $H_t$ and utility function

$$u(C, H) = \frac{(C - H)^{1-\gamma}}{1 - \gamma}$$

- Surplus consumption ratio

$$S_t = \frac{C_t - H_t}{C_t}$$

# Campbell and Cochrane (1999) – Stochastic Discount Factor

- Marginal utility is given by

$$u_c(C, H) = (C - H)^{-\gamma} = C^{-\gamma} S^{-\gamma}.$$

- The SDF is thus given by

$$M_{t,t+1} = \mathrm{e}^{-\delta} \frac{u_c(C_{t+1}, H_{t+1})}{u_c(C_t, H_t)} = \mathrm{e}^{-\delta} \left( \frac{S_{t+1}}{S_t} \frac{C_{t+1}}{C_t} \right)^{-\gamma}$$

and the log SDF

$$\begin{aligned} m_{t,t+1} &= -\delta - \gamma \log \frac{C_{t+1}}{C_t} - \gamma \log \frac{S_{t+1}}{S_t} \\ &= -\delta - \gamma(\mu_c + \nu_{t+1}) - \gamma \Delta s_{t+1} \end{aligned}$$

# Campbell and Cochrane (1999) – Risk-free Rate

- To gain intuition, assume for the moment that $\Delta s_{t+1} = \mu_s + \xi_{t+1}$, where $\xi_{t+1} \sim_{i.i.d.} \mathcal{N}(0, \sigma_s^2)$.
- Then, the log SDF is given by

$$m_{t,t+1} = -\delta - \gamma(\mu_c + \nu_{t+1}) - \gamma(\mu_s + \xi_{t+1}).$$

  and the pricing kernel is log-normally distributed.
- The risk-free rate is thus

$$\begin{aligned} r_t^f &= -\log(\mathbb{E}_t[M_{t,t+1}]) \\ &= -\mathbb{E}_t[m_{t,t+1}] - \frac{1}{2}\mathrm{var}_t[m_{t,t+1}] \\ &= \delta + \gamma\mu_c + \gamma\mu_s - \frac{1}{2}\gamma^2\sigma_c^2 - \frac{1}{2}\gamma^2\sigma_s^2 - \gamma^2\mathrm{cov}_t(\nu_{t+1}, \xi_{t+1}) \end{aligned}$$

$$r_t^f = \delta + \gamma\mu_c - \frac{1}{2}\gamma^2\sigma_c^2 + \gamma\mu_s - \frac{1}{2}\gamma^2\sigma_s^2 - \gamma^2 \mathrm{cov}_t(\nu_{t+1}, \xi_{t+1})$$

- The blue terms are standard.
  - $\delta$ represents the role of discounting.
  - $\gamma\mu_c$ represents intertemporal consumption smoothing.
  - $-\frac{1}{2}\gamma^2\sigma_c^2$ represents precautionary savings (consumption risk).
- Compared to CRRA-utility the red terms are new.
  - $\gamma\mu_s$ intertemporal consumption smoothing.
  - $-\frac{1}{2}\gamma^2\sigma_s^2 + \gamma^2\mathrm{cov}_t(\nu_{t+1}, \xi_{t+1})$ precautionary savings.

# Campbell and Cochrane (1999) – Equity Premium

- Risk premium for an asset with return $r_{t+1}$:

$$
\begin{aligned}
\mathrm{rp}_t &= \mathbb{E}_t[r_{t+1}] + \frac{1}{2}\mathrm{var}_t(r_{t+1}) - r_t^f \\
&= -\mathrm{cov}(r_{t+1}, m_{t+1}) \\
&= \gamma \mathrm{cov}(r_{t+1}, c_{t+1}) + \gamma \mathrm{cov}(r_{t+1}, s_{t+1})
\end{aligned}
$$

- The blue term is standard.
- Compared to CRRA-utility the red term reflects an additional risk-premium for the uncertainty in the habit level.
- The risk premium is higher than in the CRRA case since risk in $s$ is priced.

# Campbell and Cochrane (1999) – Surplus Consumption Dynamics

- Campbell and Cochrane (1999) specify:

$$\Delta s_{t+1} = \varphi(\bar{s} - s_t) + \lambda(s_t)\nu_{t+1}$$

- Mean reversion process with mean reversion speed $\varphi$ and mean reversion level $\bar{s}$.
- Same risk factor as consumption growth.
- Open question is how to model the sensitivity function $\lambda$.
- Notice that this indirect way to model habit ensures that surplus consumption stays positive.

# Campbell and Cochrane (1999) – Surplus Consumption Dynamics

$$\Delta s_{t+1} = \boxed{\underbrace{\varphi(\bar{s} - s_t)}_{<0}} + \lambda(s_t) \cdot v_{t+1}$$


$s_t < \bar{s} \Rightarrow$ positive drift rate

$s_t > \bar{s} \Rightarrow$ negative drift rate

$$S_t = \frac{C_t - H_t}{C_t}$$

$$s_{t+1} \nsim \mathcal{N}(\mu_s, \sigma_s^2), \quad s_{t+1} \mid \mathcal{F}_t \sim \mathcal{N}(\mu_s, \sigma_s^2)$$

- Now, the risk-free rate becomes

$$r_t^f = \delta + \gamma\mu_c - \frac{1}{2}\gamma^2\sigma_c^2 - \gamma\varphi(s_t - \bar{s}) - \frac{1}{2}\gamma^2\left(2\lambda(s_t) + \lambda(s_t)^2\right)\sigma_c^2$$

- ... and the equity premium

$$\begin{aligned} \mathrm{rp}_t &= \gamma\mathrm{cov}_t(r_{t+1}, \Delta c_{t+1}) + \gamma\mathrm{cov}_t(r_{t+1}, \Delta s_{t+1}) \\ &= \gamma\mathrm{cov}_t(r_{t+1}, \Delta c_{t+1}) + \gamma\lambda(s_t)\mathrm{cov}_t(r_{t+1}, \Delta c_{t+1}) \end{aligned}$$

- Blue terms are standard, red terms due to habit formation

# Campbell and Cochrane (1999) – Specification of $\lambda(s)$

Campbell and Cochrane (1999) impose three conditions:

1. risk-free rate is constant at $\bar{r}$.

$$\bar{r} = \delta + \gamma\mu_c - \frac{1}{2}\gamma^2\sigma_c^2 - \gamma\varphi(s_t - \bar{s}) - \frac{1}{2}\gamma^2\left(2\lambda(s_t) + \lambda(s_t)^2\right)\sigma_c^2$$

2. habit is predetermined *at* the steady state $\bar{s}$.

$$\left.\frac{\partial h}{\partial c}\right|_{s=\bar{s}} = 1 - \frac{\lambda(\bar{s})}{\mathrm{e}^{-\bar{s}} - 1} = 0$$

3. habit is predetermined *around* the steady state $\bar{s}$.

$$\left.\frac{\partial^2 h}{\partial c \partial h}\right|_{s=\bar{s}} = -\frac{\lambda'(\bar{s})(e^{-\bar{s}} - 1) + \lambda(\bar{s})\mathrm{e}^{-\bar{s}}}{(\mathrm{e}^{-\bar{s}} - 1)^2} = 0$$

# Campbell and Cochrane (1999) – Specification of $\lambda(s)$

- These conditions impose a restriction between the steady-state surplus consumption ratio and the other parameters

$$e^{\overline{s}} = \sigma_c \sqrt{\frac{\gamma}{\varphi}}.$$

- They lead to a specification of the sensitivity function

$$\lambda(s_t) = \left( e^{-\overline{s}} \sqrt{1 - 2(s_t - \overline{s})} - 1 \right) \mathbf{1}_{\{s_t \le s_{\max}\}}.$$

- Risk-free rate

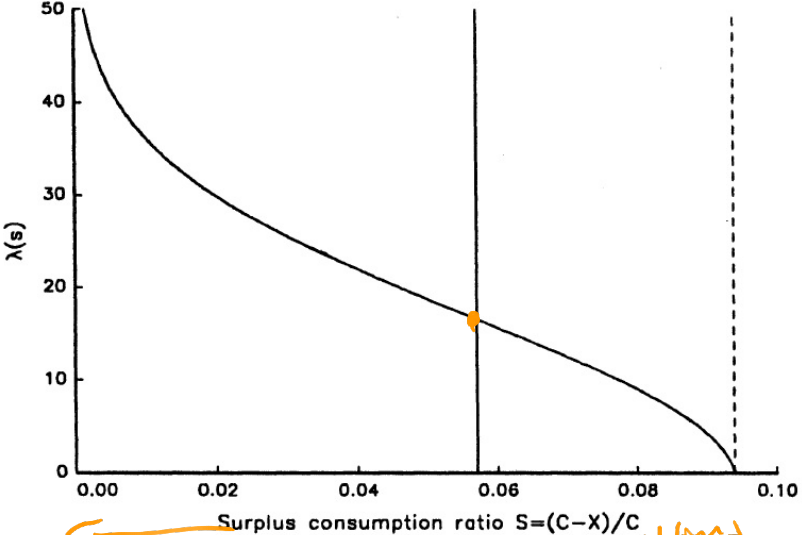
$$r_t^f = \delta + \gamma \mu_c - 0.5 \gamma \varphi$$

- Risk premium

$$\mathrm{rp}_t = \frac{\gamma}{\overline{S}} \sqrt{1 - 2(s_t - \overline{s})} \mathrm{cov}(r_{t+1}, \Delta c_{t+1})$$

| Parameter | Variable | Value |
|---|---|---|
| Assumed: | | |
| Mean consumption growth (%)* | $g = \mu_c$ | 1.89 |
| Standard deviation of consumption growth (%)* | $\sigma_c$ / | 1.50 |
| Log risk-free rate (%)* | $r^f$ | .94 |
| Persistence coefficient* | $\phi$ | .87 |
| Utility curvature | $\gamma$ | $\boxed{2.00}$ |
| Standard deviation of dividend growth (%)* | $\sigma_w$ | 11.2 |
| Correlation between $\Delta d$ and $\Delta c$ | $\rho$ | .2 |
| Implied: | | |
| Subjective discount factor* | $\delta$ | .89 |
| Steady-state surplus consumption ratio | $\bar{S}$ | $\boxed{.057}$ |
| Maximum surplus consumption ratio | $S_{\max}$ | $\boxed{.094}$ |

# Campbell and Cochrane (1999) – Calibration




bad times: vol is high

good times

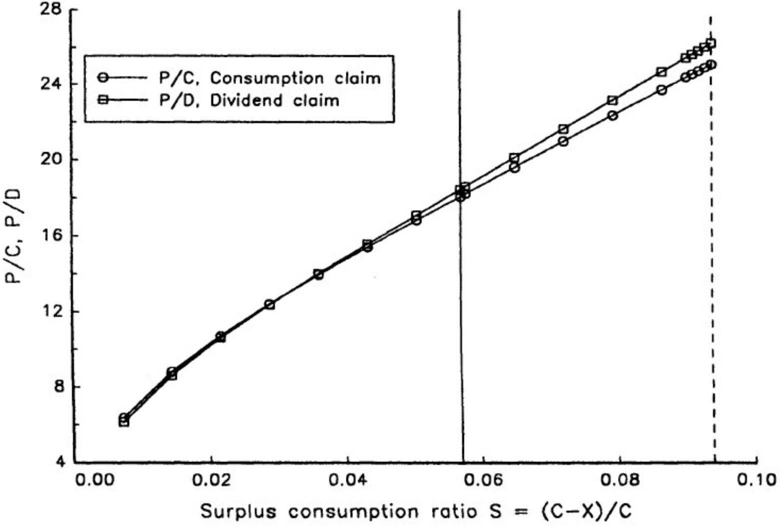
vol. is low

# Campbell and Cochrane (1999) – Asset Pricing Moments

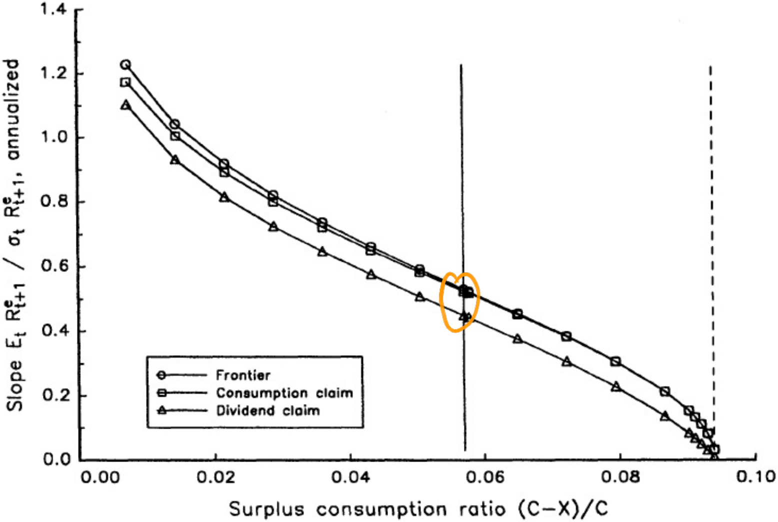
MEANS AND STANDARD DEVIATIONS OF SIMULATED AND HISTORICAL DATA

| Statistic | Consumption Claim | Dividend Claim | Postwar Sample | Long Sample |
|---|---|---|---|---|
| $E(\Delta c)$ | 1.89* | | 1.89 | 1.72 |
| $\sigma(\Delta c)$ | 1.22* | | 1.22 | 3.32 |
| $E(r^f)$ | .094* | | .094 | 2.92 |
| $E(r - r^f)/\sigma(r - r^f)$ | .43* | .33 | .43 | .22 |
| $E(R - R^f)/\sigma(R - R^f)$ | .50 | | .50 | |
| $E(r - r^f)$ | 6.64 | 6.52 | 6.69 | 3.90 |
| $\sigma(r - r^f)$ | 15.2 | 20.0 | 15.7 | 18.0 |
| $\exp[E(p - d)]$ | 18.3 | 18.7 | 24.7 | 21.1 |
| $\sigma(p - d)$ | .27 | .29 | .26 | .27 |

# Campbell and Cochrane (1999) – Price Dividend Ratio

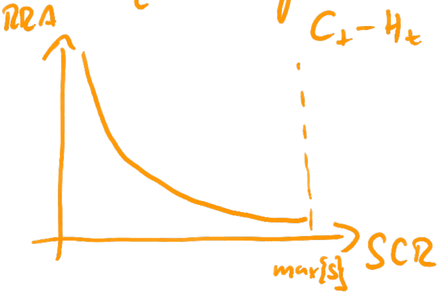


# Campbell and Cochrane (1999) – Counter-cyclical Sharpe Ratios



# Campbell and Cochrane (1999) – State-dependent Risk Aversion

$$RRA_t = \gamma \frac{C_t}{C_t - H_t} = \gamma / S_t$$




$$\left.\begin{aligned} \bar{S} &= 0.057 \\ \gamma &= 2 \end{aligned}\right\} \text{ Steady State RRA}$$

$$= \frac{2}{0.057} \approx 35$$

$$\max S = 0.094$$

$$\Rightarrow \min RRA = \frac{2}{0.094} \approx 21$$

① Agent gains utility from consumption that exceeds the habit level

$$\mathcal{U} = \frac{1}{1-\gamma}(C-H)^{1-\gamma} = \frac{1}{1-\gamma}(C\cdot S)^{1-\gamma}$$

② Agents utility from state-dependent consumption

$$\hat{C} = C\cdot S \Rightarrow \hat{C}_{t+1} = \hat{C}_t e^{\Delta c_{t+1} + \Delta s_{t+1}} = \hat{C}_t e^{\Delta \hat{c}_{t+1}}$$

$$\Delta \hat{c}_{t+1} = \mu_c + v_{t+1}\sigma_c + \varphi(\bar{s} - s_t) + \lambda(s_t) v_{t+1}$$

$$= \underbrace{\mu_c + \varphi(\bar{s} - s_t)} + (\underline{\lambda(s_t)} + 1) v_{t+1}$$