

- 1 Option Pricing in Partial Equilibrium
- 2 General Equilibrium Asset Pricing
- 3 **Habit Formation and Asset Pricing**
 - **Motivation**
 - Campbell/Cochrane Model

Motivation for Habit Formation

- The standard Lucas tree model with CRRA utility is not able to match the relevant asset pricing moments.
- More realistic preference structure involving the agent's habit level of consumption.
- Investor does not look at consumption only, but compares it to some benchmark.
- Variation in the habit level over time
 - can be used to model business cycles.
 - leads to state dependent risk aversion.
 - can lead to time-dependent variations, e.g., counter-cyclical Sharpe ratios.

- Popular choices with instantaneous utility function

$$u(C, H) = \frac{(C - H)^{1-\gamma}}{1 - \gamma}, \quad u(C, H) = \frac{(C/H)^{1-\gamma}}{1 - \gamma}.$$

- H denotes the habit level.
- Investor compares own consumption to some benchmark which can
 - ① either depend on own past consumption (internal habit)
 - ② or be some exogenous process (external habit)
- An admissible consumption strategy satisfies $C_t > H_t$ for all $t = 1, \dots, T$.

- Degree of relative risk aversion for additive habit

$$RRA(C, H) = -C \frac{u_{cc}(C, H)}{u_c(C, H)} = \gamma \frac{C}{C - H}.$$

- Risk aversion is higher in bad times, i.e., if $C - H$ is small.
- Utility index

$$U = \sum_{t=0}^T \beta^t E[u(C_t, H_t)] e^{-\delta t}$$

● Internal Habit

- Agent compares consumption today to average consumption in the past, e.g., Abel (1999)
- Habit level is moving average of past consumption

$$H_t = H_0 e^{-\beta t} + \alpha \sum_{s=1}^{t-1} e^{-\beta(t-s)} C_s$$

- Habit level dynamics

$$H_t = e^{-\beta} H_{t-1} + \alpha e^{-\beta} C_{t-1}$$

- **External Habit** is determined by consumption of other investors, e.g., Campbell and Cochrane (1999), Abel (1990).

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- Endowment economy with aggregate consumption dynamics

$$\Delta c_{t+1} = \mu_c + \nu_{t+1}$$

where the innovations $\nu_{t+1} \sim i.i.d. \mathcal{N}(0, \sigma_c^2)$

- Agent has **external** habit formation with habit level H_t and utility function

$$u(C, H) = \frac{(C - H)^{1-\gamma}}{1-\gamma}$$

- Surplus consumption ratio

$$S_t = \frac{C_t - H_t}{C_t}$$

Campbell and Cochrane (1999) – Stochastic Discount Factor

- Marginal utility is given by

$$u_c(C, H) = (C - H)^{-\gamma} = C^{-\gamma} S^{-\gamma}.$$

- The SDF is thus given by

$$M_{t,t+1} = e^{-\delta} \frac{u_c(C_{t+1}, H_{t+1})}{u_c(C_t, H_t)} = e^{-\delta} \underbrace{\left(\frac{S_{t+1}}{S_t} \frac{C_{t+1}}{C_t} \right)^{-\gamma}}$$

and the log SDF

$$\begin{aligned} m_{t,t+1} &= -\delta - \gamma \log \frac{C_{t+1}}{C_t} - \gamma \log \frac{S_{t+1}}{S_t} \\ &= \underbrace{-\delta - \gamma(\mu_c + \nu_{t+1})}_{\text{}} - \underbrace{\gamma \Delta s_{t+1}}_{\text{}} \end{aligned}$$

Campbell and Cochrane (1999) – Risk-free Rate

- To gain intuition, assume for the moment that $\Delta s_{t+1} = \mu_s + \xi_{t+1}$, where $\xi_{t+1} \sim i.i.d. \mathcal{N}(0, \sigma_s^2)$.
- Then, the log SDF is given by

$$m_{t,t+1} = -\delta - \gamma(\mu_c + \nu_{t+1}) - \gamma(\mu_s + \xi_{t+1}).$$

and the pricing kernel is log-normally distributed.

- The risk-free rate is thus

$$\begin{aligned} r_t^f &= -\log(\mathbb{E}_t[M_{t,t+1}]) \\ &= -\mathbb{E}_t[m_{t,t+1}] - \frac{1}{2}\text{var}_t[m_{t,t+1}] \\ &= \underbrace{\delta + \gamma\mu_c}_{\text{blue}} + \underbrace{\gamma\mu_s}_{\text{red}} - \underbrace{\frac{1}{2}\gamma^2\sigma_c^2}_{\text{blue}} - \underbrace{\frac{1}{2}\gamma^2\sigma_s^2}_{\text{red}} - \underbrace{\gamma^2\text{cov}_t(\nu_{t+1}, \xi_{t+1})}_{\text{red}} \end{aligned}$$

$$r_t^f = \delta + \gamma\mu_c - \frac{1}{2}\gamma^2\sigma_c^2 + \gamma\mu_s - \frac{1}{2}\gamma^2\sigma_s^2 - \gamma^2\text{cov}_t(\nu_{t+1}, \xi_{t+1})$$

- The blue terms are standard.
 - δ represents the role of discounting.
 - $\gamma\mu_c$ represents intertemporal consumption smoothing.
 - $-\frac{1}{2}\gamma^2\sigma_c^2$ represents precautionary savings (consumption risk).
- Compared to CRRA-utility the red terms are new.
 - $\gamma\mu_s$ intertemporal consumption smoothing.
 - $\gamma\sigma_s^2 + \gamma^2\text{cov}_t(\nu_{t+1}, \xi_{t+1})$ precautionary savings.

- Risk premium for an asset with return r_{t+1} :

$$\begin{aligned} \text{rp}_t &= \mathbb{E}_t[r_{t+1}] + \frac{1}{2} \text{var}_t(r_{t+1}) - r_t^f \\ &= -\text{cov}(r_{t+1}, m_{t+1}) \\ &= \gamma \text{cov}(r_{t+1}, c_{t+1}) + \gamma \text{cov}(r_{t+1}, s_{t+1}) \end{aligned}$$

- The blue term is standard.
- Compared to CRRA-utility the red term reflects an additional risk-premium for the uncertainty in the habit level.
- The risk premium is higher than in the CRRA case since risk in s is priced.

Campbell and Cochrane (1999) – Surplus Consumption Dynamics

- Campbell and Cochrane (1999) specify:

$$\Delta s_{t+1} = \varphi(\bar{s} - s_t) + \lambda(s_t)\nu_{t+1}$$

- Mean reversion process with mean reversion speed φ and mean reversion level \bar{s} .
- Same risk factor as consumption growth.
- Open question is how to model the sensitivity function λ .
- Notice that this indirect way to model habit ensures that surplus consumption stays positive.

Campbell and Cochrane (1999) – Surplus Consumption Dynamics

- Now, the risk-free rate becomes

$$r_t^f = \delta + \gamma\mu_c - \frac{1}{2}\gamma^2\sigma_c^2 - \gamma\varphi(s_t - \bar{s}) - \frac{1}{2}\gamma^2(2\lambda(s_t) + \lambda(s_t)^2)\sigma_c^2$$

- ... and the equity premium

$$\begin{aligned}rp_t &= \gamma\text{cov}(r_{t+1}, \Delta c_{t+1}) + \gamma\text{cov}(r_{t+1}, \Delta s_{t+1}) \\ &= \gamma\text{cov}(r_{t+1}, \Delta c_{t+1}) + \gamma\lambda(s_t)\text{cov}(r_{t+1}, \Delta c_{t+1})\end{aligned}$$

- Blue terms are standard, red terms due to habit formation

Campbell and Cochrane (1999) – Specification of $\lambda(s)$

Campbell and Cochrane (1999) impose three conditions:

- 1 risk-free rate is constant at \bar{r} .

$$\bar{r} = \delta + \gamma\mu_c - \frac{1}{2}\gamma^2\sigma_c^2 - \gamma\varphi(s_t - \bar{s}) - \frac{1}{2}\gamma^2(2\lambda(s_t) + \lambda(s_t)^2)\sigma_c^2$$

- 2 habit is predetermined *at* the steady state \bar{s} .

$$\left. \frac{\partial h}{\partial c} \right|_{s=\bar{s}} = 1 - \frac{\lambda(\bar{s})}{e^{-\bar{s}} - 1} = 0$$

- 3 habit is predetermined *around* the steady state \bar{s} .

$$\left. \frac{\partial^2 h}{\partial c \partial h} \right|_{s=\bar{s}} = - \frac{\lambda'(\bar{s})(e^{-\bar{s}} - 1) + \lambda(\bar{s})e^{-\bar{s}}}{(e^{-\bar{s}} - 1)^2} = 0$$

Campbell and Cochrane (1999) – Specification of $\lambda(s)$

- These conditions impose a restriction between the steady-state surplus consumption ratio and the other parameters

$$e^{\bar{s}} = \sigma_c \sqrt{\frac{\gamma}{\varphi}}.$$

- They lead to a specification of the sensitivity function

$$\lambda(s_t) = \left(e^{-\bar{s}} \sqrt{1 - 2(s_t - \bar{s})} - 1 \right) \mathbf{1}_{\{s_t \leq s_{\max}\}}.$$

- Risk-free rate

$$r_t^f = \delta + \gamma \mu_c - 0.5 \gamma \varphi$$

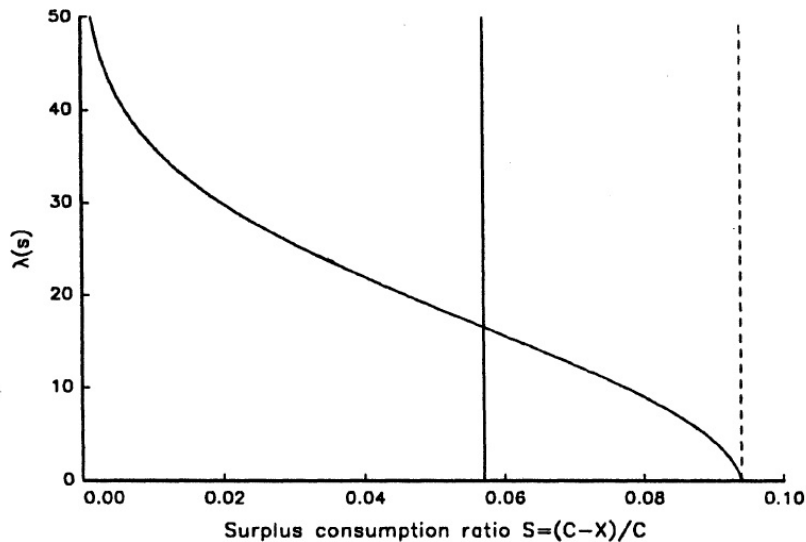
- Risk premium

$$\text{rp}_t = \frac{\gamma}{\bar{S}} \sqrt{1 - 2(s_t - \bar{s})} \text{cov}(r_{t+1}, \Delta c_{t+1})$$

Campbell and Cochrane (1999) – Calibration

Parameter	Variable	Value
Assumed:		
Mean consumption growth (%)*	\bar{g}	1.89
Standard deviation of consumption growth (%)*	σ	1.50
Log risk-free rate (%)*	r^f	.94
Persistence coefficient*	ϕ	.87
Utility curvature	γ	2.00
Standard deviation of dividend growth (%)*	σ_w	11.2
Correlation between Δd and Δc	ρ	.2
Implied:		
Subjective discount factor*	δ	.89
Steady-state surplus consumption ratio	\bar{S}	.057
Maximum surplus consumption ratio	S_{\max}	.094

Campbell and Cochrane (1999) – Calibration

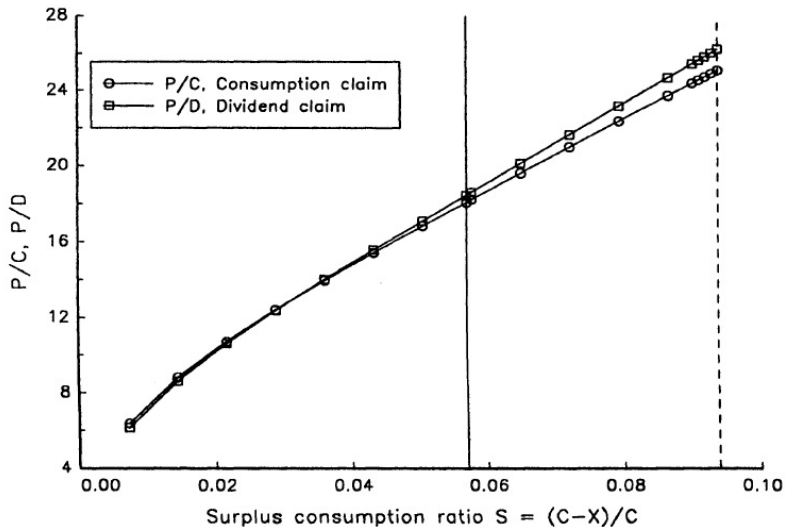


Campbell and Cochrane (1999) – Asset Pricing Moments

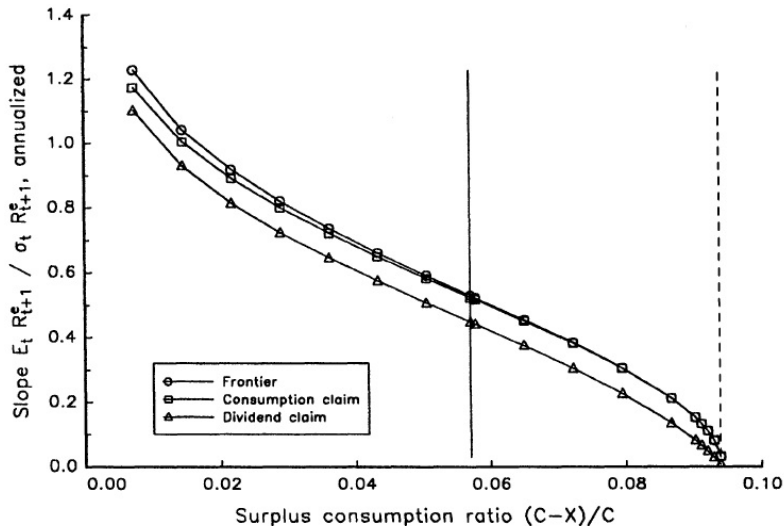
MEANS AND STANDARD DEVIATIONS OF SIMULATED AND HISTORICAL DATA

Statistic	Consumption Claim	Dividend Claim	Postwar Sample	Long Sample
$E(\Delta c)$	1.89*		1.89	1.72
$\sigma(\Delta c)$	1.22*		1.22	3.32
$E(r^f)$.094*		.094	2.92
$E(r - r^f) / \sigma(r - r^f)$.43*	.33	.43	.22
$E(R - R^f) / \sigma(R - R^f)$.50		.50	
$E(r - r^f)$	6.64	6.52	6.69	3.90
$\sigma(r - r^f)$	15.2	20.0	15.7	18.0
$\exp[E(p - d)]$	18.3	18.7	24.7	21.1
$\sigma(p - d)$.27	.29	.26	.27

Campbell and Cochrane (1999) – Price Dividend Ratio



Campbell and Cochrane (1999) – Counter-cyclical Sharpe Ratios



Campbell and Cochrane (1999) – State-dependent Risk Aversion

Campbell and Cochrane (1999) – Alternative Interpretation