Advanced Financial Economics I – Part 1: Discrete Time Models –

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Agenda

- 1 Option Pricing in Partial Equilibrium
- 2 General Equilibrium Asset Pricing
- Habit Formation and Asset Pricing
 - 4 Recursive Utility
- 5 Long-Run Risk and Asset Pricing
- 6 Disaster Risk and Asset Pricing

7 Heterogeneity

Agenda



- 2 General Equilibrium Asset Pricing
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- Disaster Risk and Asset Pricing

Heterogeneity

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Primary Securities

- Probability space $(\Omega, \mathcal{A}, \mathbb{P})$
- One period: t = 0, T.
- Asset prices are **exogenously** given by a (n + 1)-dimensional positive process.
- Money market account S^0 with interest rate r

$$S_0^0 = 1, \qquad S_T^0 = S_0^0(1+r)$$

• *n* primary securities (stocks)

$$S = (S_t)_{t=0,T}, \qquad S_t = (S_t^0, \dots, S_t^n)^ op$$

where $S_0 \in \mathbb{R}^{n+1}$ and S_T is a (n+1)-dimensional random vector with finitely many possible outcomes.

A (contingent) claim guarantees a payoff C_T at T.

• A **Forward Contract** obligates the holder to buy (or sell) an asset at a fixed price *K* at a predetermined date *T*. Payoff profile:

$$C_T = S_T - K$$

• A **Call Option** offers the right (but not the obligation) to *buy* an asset at a fixed price K on or up to a specified date T. Payoff profile:

$$C_T = \max\{0, S_T - K\}$$

• A **Put Option** offers the right (but not the obligation) to *sell* an asset at a fixed price K on or up to a specified date T. Payoff profile:

$$C_T = \max\{0, K - S_T\}$$

Contingent Claims



Trading Strategy

- A trading strategy $\varphi = (\varphi^0, \dots, \varphi^n)^\top \in \mathbb{R}^{n+1}$ is a (n+1)-dimensional vector.
- φ^0 : number of bonds.
- $\varphi^0 S_t^0$: money amount invested in the bond.
- φ^i : number of shares of stock $i = 1, \ldots, n$.
- $\varphi^i S_t^i$: money amount invested in stock i = 1, ..., n.

Financial Wealth

Financial Wealth $X_t = X_t^{\varphi}$ at time t = 0, T is given by the portfolio value of the trading strategy

$$X_t = \varphi^\top S_t = \sum_{i=0}^n \varphi^i S_t^i.$$

It describes the portfolio value for an investor using the trading strategy φ .

Pricing of Contingent Claims

Definition

• A trading strategy φ is an **arbitrage opportunity** if

$$X_0^{\varphi}=0, \qquad X_T^{\varphi}\geq 0, \qquad \mathbb{P}(X_T^{\varphi}>0)>0.$$

- A model is arbitrage-free if no arbitrage opportunities exist.
- A claim *C* is **attainable** if a trading strategy φ exists such that $X_T^{\varphi} = C_T$. Such a strategy is called a **replication strategy** or **hedging strategy**.
- A financial market is **complete** if and only if all contingent claims are attainable.
- We only consider arbitrage-free, but not necessarily complete models.
- In models with arbitrage opportunities, very strange things can happen.

Theorem: Law of One Price

Suppose the market is arbitrage-free.

() For an attainable claim C with hedging strategy φ ,

 $C_0 = X_0^{\varphi} \qquad \begin{array}{c} price \\ = prifield \ v \ mathematical \\ of \ mathematical \\ of$

3 If
$$X_T^{\varphi} = X_T^{\psi}$$
 for trading strategies φ and ψ , then

$$X_0^{\varphi} = X_0^{\psi}.$$

Proof: LOP

Suppose thre is another ablinge-fer price $C'_{o} > C_{o} = X''_{o}$ at $\lim_{t=0}$; shahjy Canside the following $m = \frac{C_0}{2} > 1$ · Buy m-times the repl. p-lfto · Short-sell the chim -mXo C, $\overline{C_{o}^{\prime}}-m\chi_{o}^{\prime\prime}=0$ AL HET · gain m-line the contrast dwg · Liquidite Short position + CTM - CT Advanced Financial Economics Ninter Term 2021/22 10/193

Proof: LOP

 $(m-1)C_T > 0$ こて => arbbype oppubnily => Carbondichin => altrup-free price is unifue (2) Asson Xo > Xo Strulegy: "Buy sheep ad sell expressive" $m = \frac{X_0^4}{X_0^4} > 1$ Buy m-kmi $4: -mX_0^4$ T:at T: $-X_{T}^{\varphi}+mX_{T}^{\varphi}=(m-1)X_{T}^{\varphi}>0$ palfalto volue

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Example: Binomial Model – Model Setup

We consider the simplest possible model with only two assets

- Money Market Account $B_1 = B_0(1 + r)$
- Stock $S_1 = S_0(1+y)$, where

$$p = \mathbb{P}(y = \underline{u}) = 1 - \mathbb{P}(y = \underline{d}) \in (0, 1)$$

• u > r > d. if his and the is wolked the Soll+d) • Contingent claim with payoff:

$$C_T = c_u \mathbf{1}_{\{y=u\}} + c_d \mathbf{1}_{\{y=d\}}$$

• Example: Call Option with strike price $S_0(1+u) > K > S_0(1+d)$:

$$c_u = S_0(1+u) - K, \qquad c_d = 0$$

Example: Binomial Model - Replication

- A replication strategy has to satisfy the following linear system $\begin{aligned}
 \left(\mathcal{G} = \left(\mathcal{G}_{0}, \mathcal{G}_{0} \right) & c_{u} = \varphi^{0} S_{0}^{0} (1+r) + \varphi^{1} S_{0}^{1} (1+u) \\
 c_{d} = \varphi^{0} S_{0}^{0} (1+r) + \varphi^{1} S_{0}^{1} (1+d) \end{aligned}\right)
 \end{aligned}$
 - This system has a unique solution

$$\underline{\varphi^0 S_0^0} = \frac{c_d(1+u) - c_u(1+d)}{(1+r)(u-d)}, \qquad \underline{\varphi^1 S_0^1} = \frac{c_u - c_d}{u-d}.$$

• The price of the claim is thus

$$C_0 = X_0^{\varphi} = \varphi^0 S_0^0 + \varphi^1 S_0^1 = \frac{q}{1+r} c_u + \frac{1-q}{1+r} c_d$$

where $q = \frac{r-d}{u-d} \mathcal{E} (0, 1)$

Example: Binomial Model – Risk-Neutral Probability

• The price has thus the following representation

$$C_0 = \mathbb{E}^{\mathbb{Q}} \Big[\frac{C_T}{1+r} \Big]$$

where ${\ensuremath{\mathbb Q}}$ is a probability measure with

$$\mathbb{Q}(Y=u)=q=1-\mathbb{Q}(Y=d).$$

- Such a measure is called a risk-neutral measure.
- Under Q, prices can be calculated as expected discounted cashflows.
- There is no risk-premium involved.
- Asset prices satisfy the pricing relation

$$S_0^i = \mathbb{E}^{\mathbb{Q}}\left[\frac{S_T^i}{1+r}\right]. \qquad i = 0, \dots, N$$

In the binomial example, we constructed a probability measure $\mathbb Q$ that relates in a certain sense to $\mathbb P.$

Definition

Two probability measures \mathbb{P} and \mathbb{Q} are said to be equivalent if both measures possess the same null sets, i.e., for all events $A \in A$

$$\mathbb{P}(A) = 0 \iff \mathbb{Q}(A) = 0.$$

We use the notation $\mathbb{P}\sim\mathbb{Q}$ for equivalent probability measures.

Change of Measure – Equivalent Probability Measures

The following theorem states how to switch between two equivalent probability measures.

Theorem: Radon-Nikodym

Let $\mathbb{P} \sim \mathbb{Q}$ denote two equivalent probability measures, then there exists a unique (a.s.), positive random variable $Z = \frac{d\mathbb{Q}}{d\mathbb{P}}$ such that $\boldsymbol{\gamma}$

$$\mathbb{E}^{\mathbb{Q}}[Y] = \mathbb{E}^{\mathbb{P}}[ZY]$$
 $\mathbb{E}^{\mathbb{P}}[Y] = \mathbb{E}^{\mathbb{Q}}\Big[rac{Y}{Z}\Big]$

EZT

for all random variables $Y : \Omega \to \mathbb{R}$.

In our example, the process Z is just a binomial random variable given by $= \rho \begin{pmatrix} q \\ r \end{pmatrix} + (1-p) \begin{pmatrix} 1-q \\ r \end{pmatrix} \int d^{d} \\ p \\ = \beta \begin{pmatrix} u \\ r \end{pmatrix} = \frac{q}{p}, \qquad Z^{d} = \frac{1-q}{1-p} = \frac{f^{2}}{q} \cdot \gamma^{2} + (1-q) \gamma^{d} \\ = \frac{f^{2}}{q} \cdot \gamma^{2} + (1-q) \gamma^{d}$ Christen Hambel Advanced Financial Economics 1 Winter Term 2021/22 16/193

Definition

A probability measure $\mathbb{Q} \sim \mathbb{P}$ is called a **risk-neutral measure** or an **equivalent martingale measure** if and only if

$$S_0^i = \mathbb{E}^{\mathbb{Q}}\Big[rac{S_T^i}{1+r}\Big]$$

for all assets $i = 0, \ldots, n$.

- Under a risk-neutral measure, one can calculate asset prices as a discounted expected value.
- The pricing problem thus collapses to the calculation of an expectation.

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Theorem: Risk-Neutral Pricing

Suppose the market is arbitrage-free. Let \mathbb{Q} be a risk-neutral probability measure and let *C* be an attainable claim with hedging strategy φ . Then its unique arbitrage-free price_is given by

$$C_0 = X_0^{\varphi} = \mathbb{E}^{\mathbb{Q}} \Big[\frac{C_T}{1+r} \Big]$$

Proof: Risk-Neutral Pricing

· Undymiess and Co = Xo hus adready been prover. (LOP) • 2nd eq: $C_T = Z_1 \varphi^i S_T = \varphi^i S_T$ $\mathbb{E}^{\mathbb{Q}}\left[\frac{C_{T}}{1+r}\right] = \frac{1}{1+r} \mathbb{E}^{\mathbb{Q}}\left[\varphi^{T}S_{T}\right]$ $= \frac{1}{1+\tau} \varphi^{\mathsf{T}} \mathbf{F}^{\mathsf{C}} [S_{\mathsf{T}}]$ = $\varphi^T S_o = \overline{Z} \varphi_i \cdot S_o^i$ $= \chi_0^{\varphi} = C_0$ 19/193

- In general, \mathbb{Q} is more pessimistic than \mathbb{P} .
- In reality, most investors are risk-averse and demand for a positive risk premium.
- The pricing relation is thus

$$\mathbb{E}^{\mathbb{Q}}\left[\frac{C_{T}}{1+r}\right] = \mathbb{E}^{\mathbb{P}}\left[\frac{C_{T}}{1+r+\mathrm{rp}_{T}}\right]$$

- Notice that $\frac{1}{1+r+rp_T} < \frac{1}{1+r}$.
- If both expectations are equal, the risk-neutral measure puts more weight on bad events.
- In the binomial example $Z^u = \frac{q}{p} < 1$, $Z^d = \frac{1-q}{1-p} > 1$.

q 1-p

Example: Binomial Model – Stochastic Discount Factor

• Recall: The price of the claim is

$$C_{0} = \frac{q}{1+r}c_{u} + \frac{1-q}{1+r}c_{d}$$

= $p_{p(1+r)}q_{u} + (1-p)\frac{1-q}{(1-p)(1+r)}c_{d}$.

• Therefore, the price can be expressed as

$$C_0 = \mathbb{E}^{\mathbb{P}}\Big[C_T \cdot M_T\Big]$$

where $M_T^u = rac{q}{
ho(1+r)}$ and $M_T^d = rac{1-q}{(1ho)(1+r)}$, i.e., $M_T = rac{Z}{1+r}$.

- Such a random variable M_T is called a **stochastic discount factor** (SDF) or **pricing kernel**.
- Using the SDF, one can calculate asset prices under \mathbb{P} .

Definition

A random variable M_T is called a **stochastic discount factor** or a **pricing kernel** if and only if

$$S_0^i = \mathbb{E}^{\mathbb{P}}\Big[S_T^i \cdot M_T\Big]$$

for all assets $i = 0, \ldots, n$.

- Again, the pricing problem collapses to the calculation of an expectation.
- But: given a SDF, you can price under \mathbb{P} instead of \mathbb{Q} .
- In the following, we use the notation $\mathbb{E}[\cdot] = \mathbb{E}^{\mathbb{P}}[\cdot]$.

Stochastic Discount Factor and the Risk-free Rate

 $\overbrace{S_0^0} = \mathbb{E} \left[S_T^0 \cdot M_T \right]$

 $S_0^0 = \mathbb{E} \left[S_0^0 \cdot M_T \right] (1+r)$

 $| = \mathbb{E}[\mathbf{n}_{T}] \cdot (1+r)$

• From the definition of the SDF, it follows that

• Therefore,

$$\frac{1}{1+r} = \mathbb{E}[M_{\tau}] \qquad e^{-\tau_c} = E[\eta_{\tau}]$$

or

$$r = \frac{1}{\mathbb{E}[M_T]} - 1. \qquad \mathbf{\Gamma}_{\mathcal{C}} = - \ln \mathcal{F}(\mathbf{n})$$

• This relation holds in very complicated models.

 $S_T^o = S_0^o(1+r)$

Theorem: Stochastic Discount Factor

Suppose the market is arbitrage-free. Let M be a stochastic discount factor and let C be an attainable claim with hedging strategy φ . Then its unique arbitrage-free price is given by

$$C_0 = X_0^{\varphi} = \mathbb{E}\Big[C_T \cdot M_T\Big].$$

Given a risk-neutral measure $\mathbb{Q},$ the SDF can be expressed as

$$M_T = \frac{1}{1+r} \frac{\mathrm{d}\mathbb{Q}}{\mathrm{d}\mathbb{P}},$$

i.e., it reflects both discounting with the risk-free interest rate and a change of measure from $\mathbb P$ to $\mathbb Q.$

Proof: Stochastic Discount Factor

 $|E^{\mathbb{P}}| \eta_{T} C_{T}] = E^{\mathbb{P}}[\eta_{T} \varphi^{T} S_{T}]$ = $\varphi^T E^{\mathbb{P}} [n_T S_T]$ $= \varphi^T \cdot \varsigma_0 = \chi_0^{\varphi} = C_0$

$$M_{T} = \frac{\epsilon_{T}}{1+r}$$

$$\mathbb{E}^{\mathbb{P}}\left(\frac{2\tau}{1+r}C_{T}\right) = \frac{1}{1+r} \mathbb{E}^{\mathbb{P}}\left(\frac{2\tau}{2\tau}C_{T}\right)$$

$$= \frac{1}{1+r} \mathbb{E}^{\mathbb{Q}}\left[C_{T}\right] = C_{O}\Omega$$

Example: Put-Call Parity

 $C_{+} + \frac{\kappa}{(1+r)^{T-+}} = P_{+} + S_{+}$ Assume that the PCP is vide led. $C_{+} \stackrel{K}{=} \frac{1}{(1+r)^{T-t}} > P_{+} + S_{+}$ Prov Buy chop and sell expressive $m = \frac{C_{+} + \frac{k}{(1+r)^{T-+}}}{5}$ $P_{+} + S_{+}$ · Buy m- times a pulphic custify of pat-optim as stock · Short sell Call + K Bunds with mut T -m(P+S)+ C4 + K (170)-+

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Example: Put-Call Parity

 $a + T: \quad (1) \quad S_T > K \quad (=) \quad C_T = S_T - K$ $P_T = O$ us ST · Sell the stores: · Uznidete the shut $-(S_T-K)-K_I$ (m-1) S+ >0

sell the stock/put put fliv: m(k-ST) + m·ST
 lique duk the bond position: -K
 (m-1)K >0

Agenda



- Multi-period Model
- 2 General Equilibrium Asset Pricing
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- Disaster Risk and Asset Pricing

Heterogeneity

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Primary Securities in Discrete Time

- Probability space (Ω, A, ℙ) with filtration F = (F_t)_{t=0,...,T} modeling information.
- Trading dates: $t = 0, \ldots, T$
- Asset prices are **exogenously** given by a (n + 1)-dimensional positive adapted process.
- Money market account S^0 with interest rate r_t

$$S_0^0 > 0, \qquad S_t^0 = S_{t-1}^0(1+r_t)$$

• *n* Primary securities (stocks)

$$S = (S_t)_{t=0,\dots,T}, \qquad S_t = (S_t^0,\dots,S_t^n)^ op$$

where S is a (n + 1)-dimensional adapted process.

Sigma-Algebras, Information, and Conditional Expectations

$$\frac{+0}{10} \qquad (\frac{+1}{10}) \qquad (\frac{+1}{10}) \qquad (\frac{121}{10}) \qquad (\frac{121}{1$$

Sigma-Algebras, Information, and Conditional Expectations

Infenden is modeled as segurce of J- Myrtunes an increasing $\mathcal{X}_{o} = \{ \Omega, \emptyset \}$ $\mathcal{V}_{n} = \{ \Omega, \phi, \xi w_{n}, w_{1} \}, \{ w_{3}, w_{4} \} \}$ $\{w_1, w_3\} \notin \mathcal{X}_1$ F2 = P(SL) AEF2 mens hut we can decide at Him t if WEA

Sigma-Algebras, Information, and Conditional Expectations Nahred fillention : It is generated by the observed slock proces and shale variables. <u>Adupted process</u> if for every were X_+ is known at the t. Conditional expectations
$$\begin{split} \overline{E}_{t}[X] &= I\overline{E}[X | \mathscr{X}_{t}] , I\overline{E}_{o}[X] = I\overline{E}[X] \\ I\overline{E}_{t}[S_{2}] &= S P S^{nm} + (I-p) S^{nd}, \ \omega \in S_{u_{1}, u_{2}} \\ P \cdot S^{du} + (I-p) S^{dd}, \ \omega \in S_{u_{3}, u_{3}} \end{split}$$

Trading in Discrete Time

• A trading strategy

$$\varphi = (\varphi_t)_{t=0,\dots,T}, \qquad \varphi = (\varphi_t^0,\dots,\varphi_t^n)^\top$$

is a (n+1)-dimensional adapted process.

- φ_t^0 : number of bonds held in (t, t+1].
- φ_t^i : number of shares of stock i = 1, ..., n held in (t, t+1].

Financial Wealth

Financial Wealth $X_t = X_t^{\varphi}$ at time t is given by

$$X_t = \varphi_t^\top S_t = \sum_{i=0}^n \varphi_t^i S_t^i.$$

We consider an investor with the following properties:

- No exogenous income or expenses.
- Trading does not cause transaction costs or taxes.
- Changes in wealth only caused by changes in asset prices.

Definition A trading strategy φ is self-financing if $X_t = \varphi_t^\top S_t = \varphi_{t-1}^\top S_t.$

• Wealth after trading equals wealth before trading.

Trading in Discrete Time



Definition

• A trading strategy φ is an **arbitrage opportunity** if

$$X_0^{arphi}=0, \qquad X_T^{arphi}\geq 0, \qquad \mathbb{P}(X_T^{arphi}>0)>0.$$

- A model is arbitrage-free if no arbitrage opportunities exist.
- A claim C is **attainable** if there exists a <u>self-financing</u> trading strategy φ such that $X_T^{\varphi} = C_T$. Such a strategy is called a **replication strategy** or **hedging strategy**.
- A financial market is **complete** if and only if all contingent claims are attainable.

Pricing of Contingent Claims

- A multi-period model consists of a sequence of single-period models.
- A multi-period model is arbitrage-free if all single-period models are arbitrage-free (Delbean and Schachermeyer, 2006).
- All relevant pricing relations carry over.
- We denote the discounted asset prices by \widetilde{S}^i , i.e.,

$$\widetilde{S}_t^i = rac{S_t^i}{S_t^0}.$$

Definition

A stochastic process
$$X = (X_t)_{t=0,...,T}$$
 is called a \mathbb{P} -martingale if
$$\mathbb{E}^{\mathbb{P}}[X_{t+1} \mid \mathcal{F}_t] = X_t.$$

Examples for Martingales

a gutubles weath is a mantipuli if all gunes are fair L'on averype, wealth remains cusht. (3) IE (X+r, 18+) Z X+ subunkyulu / sopumett-pule (b) protestime Shad B93

Examples for Martingales



Definition

• A probability measure $\mathbb{Q} \sim \mathbb{P}$ is a **risk-neutral measure** or an equivalent martingale measure (EMM) if \tilde{S}^i is a martingale under \mathbb{Q} for all $i = 0, \ldots, n$, i.e., So'=E

$$\widetilde{S}_t^i = \mathbb{E}_t^{\mathbb{Q}}[\widetilde{S}_{t+1}^i].$$

• A non-negative stochastic process $M = (M_t)_{t=0,...,T}$ is called a stochastic discount factor or a pricing kernel, if MS^i is a martingale under \mathbb{P} for all $i = 0, \ldots, n$, i.e.,

$$S_t^i = \mathbb{E}_t[S_{t+1}^i M_{t+1}] rac{1}{M_t}.$$

Mt St = EL [Sun Muth]

Theorem: Pricing in Discrete Time

Assume that the market is free of arbitrage. Let C be an attainable claim with hedging strategy φ .

• Suppose that \mathbb{Q} is a risk-neutral measure. Then, the unique arbitrage-free price of *C* is

• Suppose that *M* is a stochastic discount factor. Then, the unique arbitrage-free price of *C* is

 $C_t = \mathbb{E}_t^{\mathbb{Q}} \Big[\frac{C_T}{S_T^0} \Big] S_t^0.$

$$C_t = \mathbb{E}_t [C_T M_T] \frac{1}{M_t}. \qquad \gamma_0 = 1$$

First Fundamental Theorem of Asset Pricing

The following are equivalent

• The market is free of arbitrage.

2 There exists a risk-neutral probability measure $\mathbb{Q} \sim \mathbb{P}$.

Second Fundamental Theorem of Asset Pricing

Suppose the market is free of arbitrage. The following are equivalent

- The market is complete.
- 2 There exists a unique risk-neutral probability measure $\mathbb{Q} \sim \mathbb{P}$.

Proof: FTAP 1

Assume Q~P is an Emm Y is an artitrary adapted, st. trady shahpy $\frac{\chi^{\varphi}}{S^{\circ}} \quad is \quad \alpha \quad Q - M_{\alpha}H_{\beta}h_{\beta}$ Suppose that φ is an alther oppositely $\chi_{0}^{\varphi} = 0$, $\underline{H}(\chi_{T}^{\varphi} \ge 0) = 1$ $QP(X_{\tau}^{4} > 0) > 0$

Proof: FTAP 1

 $\mathbb{E}^{\mathbb{Q}}\left(\frac{X_{\tau}}{S_{\tau}}\right) = \frac{1}{S_{\tau}^{\circ}}\mathbb{E}^{\mathbb{Q}}\left[X_{\tau}\right]$ $=\chi_{a}^{\mu}=0$ => $IE^{Q}[X_{T}^{\psi}] = 0$ { } Cubriddellin XTZO) => 4 cannot be Q(XT > 0) >0 * an allhyr oppublity => market is free of

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- Existence and uniqueness of risk-neutral probability measure are equivalent to existence and uniqueness of stochastic discount factor.
- In continuous time, the easy directions of the FTAPs still hold, i.e.,
 - **(**) Existence of risk-neutral probability measure \Rightarrow No arbitrage.
 - Oniqueness of risk-neutral probability measure Market completeness.
- For the converse directions, one has to replace the concepts of no arbitrage and EMM by somewhat more involved concepts (NFLVR, ELMM, $E\sigma$ MM), see Delbean and Schachermeyer (1994, 1998).

Example: Cox, Ross, Rubinstein (1979)

• The CRR model extends the binomial model.

• Bond price:

$$B_t = (1 + r)^t$$

Stock price:

$$S_t = S_0 \prod_{t=1}^T (1+y_t)$$

where y_t are iid with

$$p = \mathbb{P}(y_1 = u) = 1 - \mathbb{P}(y_1 = d)$$

- It is used in practice to approximate continuous-time models.
- One can show that the solution converges to the Black-Scholes formula as the number of time steps increases.

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Advanced Financial Economics I

Example: Cox, Ross, Rubinstein (1979)

- We have shown that the one-period model is free of arbitrage if u > r > d.
- Therefore, the CRR model is free of arbitrage if u > r > d and thus a risk-neutral measure exists.
- For a given claim one can find a replicating strategy φ by solving a system of two linear equations at each node of the event tree.
- Under the NA condition u > r > d these systems have unique solutions.
- The solutions provide the replicating strategy in the corresponding state at the corresponding time.
- Consequently, the model is complete and the risk-neutral measure is unique.

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Arbitrage Opportunity in CRR

were Wolahd Suppose nsrsd (u < d < r) e.g. $= C_0 = \frac{F}{1+r} C_n$ $q = \frac{r-d}{n-b}$ 1-9>1 => there is no Ash-weekel prelability => the under Contains or bitige opportunities

Completeness

 $C_T = q_1 S^n + q_2 \mathcal{B}$ $C_T = \varphi_1 S^m + \varphi_2 B$ $C_{\pm}^{d} = \varphi_{1}S^{d} - \varphi_{2}B$ => club connet be replicited => murbet is incomplete Kule of Hund m asset A mode with shuks and it is free of why and is compute of