# Capital Markets and Asset Pricing 

Goethe Business School
Summer Term 2022

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## Mock Exam

- The exam is open book, i.e., any dead objects may be used.
- Any kind of communication with other persons will be considered cheating.
- The working time is 90 minutes.
- The exam consists of three problems. The maximum number of points in every problem is 30 , so that the maximum number of total points in the exam is 90 .
- There are two types of problems:

1. Problems with a text entry field

You will answer those questions by writing a text.
2. Problems with a number input box

Input box in LPlus for $x x$


These boxes will be corrected automatically in LPlus. If your answer is correct, you will get the points. You will also have the opportunity, but not the obligation to explain your calculations at the end of each problem. I will check your explanations only if your solution in the box is wrong. In this case, I can give you some points if your documentation is understandable and evident.

Problem 1 (Bond Pricing) Assume continuous compounding throughout the problem. You observe the following data for bonds with annual payments and $100 \$$ notional.

| Type | Time-to-maturity (years) | Coupon rate | Price |
| :--- | :---: | :---: | :---: |
| Bond A | 1 | $3 \%$ | 101.98 |
| Bond B | 2 | $4 \%$ | 107.55 |
| Bond C | 3 | $5 \%$ | 110.31 |

(a) Calculate the zero rates $r(1), r(2)$, and $r(3)$.

Input box in LPlus for $r(1)$


Input box in LPlus for $r(2)$

Input box in LPlus for $r(3)$
(b) Calculate the forward rates $f(0), f(1)$, and $f(2)$.

Input box in LPlus for $f(0)$


Input box in LPlus for $f(1)$


Input box in LPlus for $f(2)$


You observe another bond (Bond D) with a yield-to-maturity of $3 \%$, time-to-maturity of two years, $2.5 \%$ coupon rate, and a notional of $100 \$$.
Assume that you hold a $100 \mathrm{~m} \$$ portfolio consisting of Bond B and Bond D, where the ratio of bond weights of Bond B to Bond D is three to two, and we impose no-short-saleconstraints.
(c) Calculate the duration and convexity of your bond portfolio.
(i) Calculate the portfolio weight of Bond B

Input box in LPlus for $w_{B}$
(ii) Calculate the yield-to-maturity of bond B


Input box in LPlus for $y_{B}$

(iii) Calculate the duration of both bonds

Input box in LPlus for $D_{B}$


Input box in LPlus for $D_{D}$

(iv) Calculate the convexity of both bonds
(4 credits)
Input box in LPlus for $\Gamma_{B}$


Input box in LPlus for $\Gamma_{D}$

(v) Calculate the portfolio duration and convexity (2 credits) Input box in LPlus for $D_{P}$

Input box in LPlus for $\Gamma_{P}$

(d) Using both duration and convexity, how does the value of your portfolio change if the yield increases by $0.5 \%$.
If you were unable to solve problem (c), use $D_{P}=2$, and $\Gamma_{D}=3.5$.
Input box in LPlus for $\Delta P$


Problem 2 (State Pricing and Credit Risk) Consider a firm with assets worth EUR 10M today. The firm has issued a single debt instrument, namely a zero coupon bond maturing in 2 years with a face value of EUR 9M. The firm value evolves according to the following tree diagram:


The discretely compounded risk-free rate is $r=2 \%$.
(a) Determine the state prices $\pi_{u}$ and $\pi_{d}$ of the two states, and the corresponding risk-neutral probabilities.
(8 credits)
Input box in LPlus for $\pi_{u}$

Input box in LPlus for $\pi_{d}$
Input box in LPlus for $q_{u}$
Input box in LPlus for $q_{d}$
Input box in LPlus for $q_{u}$
Input box in LPlus for $q_{d}$

(b) Determine the payoff of equity if the boom state materializes. What is the current value of equity?
Input box in LPlus for $E^{u}$


Input box in LPlus for $E_{0}$

(c) Determine the current value of debt, the debt-to-equity ratio, and the loss given default.
Input box in LPlus for $D_{0}$


Input box in LPlus for $\ell$


Input box in LPlus for $L G D$
(d) Calculate the credit spread, i.e., the yield spread between the corporate bond and the risk-free rate.
(4 credits)
Input box in LPlus for $s(1)$
(e) Explain verbally how the yield spread is related to probability of default. ( 6 credits)

Problem 3 (CML vs. SML) Consider an investment fund, which invests two thirds of its assets in the market portfolio and one third in the risk-free asset. The market's expected return is $7 \%$ and the risk-free rate is $1 \%$. The market volatility is $18 \%$.
(a) Determine the fund's expected return and volatility.

Input box in LPlus for $\mu_{P}$


Input box in LPlus for $\sigma_{P}$
(b) Is this fund an efficient portfolio? Explain your answer.

$\square$
(c) What is the fund's expected return according to the CML? Explain your answer.
Input box in LPlus for $\mu_{P}$

(d) What is the fund's beta and the correlation to the market portfolio? ( 5 credits)

Input box in LPlus for $\rho_{P, M}$


Input box in LPlus for $\beta_{P}$

(e) Can the fund be priced using the security market line? Explain briefly. (3 credits)

Suppose now you observe another investment fund $I$, with expected return $\mu_{I}=9 \%$.
(f) Determine the asset's beta.

Input box in LPlus for $\beta_{I}$

(g) After a thorough analysis you find out that this investment fund is indeed an efficient portfolio. What is its volatility?

Input box in LPlus for $\sigma_{I}$


