Life Insurance
– Lecture Parts III and IV –

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Spring Term 2023

Course Information

- Lecturers:
  - Feiko Drost (I: micro longevity risk and II: interest rate risk)
  - Christoph Hambel (III: macro longevity risk and IV: all risks combined)
  - Henk Keffert (tutorials)

- The second half of this course ...
  - ... provides an introduction to macro longevity risk and to applications in actuarial science that combine all types of risk.
  - ... directly builds upon the first half and does not require any additional pre-knowledge.

- Grading:
  - Exam 70%
  - Two Assignments (15% each)

What to expect?

- What can you expect from me? I will...
  - ... timely provide the learning material on Canvas
  - ... also upload the slides with hand-written complements (some parts of the slides are intentionally blank)
  - ... illustrate the lecture by examples
  - ... provide a lot of problems to practice the material
  - ... be available for questions
- What will I expect from you? You should ...
  - ... be well-prepared when you come to the lecture
  - ... actively participate in the lecture
  - ... take the opportunity and ask me questions during the classes

Preliminary Schedule

Please notice that the plan can change!

- Tue, 11.04.2023, 08:45, WZ105
- Tue, 18.04.2023, 08:45, WZ105
- Wed, 19.04.2023, 08:45, CUBE 221 (tutorial)
- Tue, 25.04.2023, 08:45, WZ105
- Wed, 26.04.2023, 08:45, CUBE 221 (tutorial)
- Tue, 09.05.2023, 08:45, WZ105
- Thu, 11.05.2023, 12:45, CZ05
- Tue, 16.05.2023, 08:45, WZ105 (tutorial)
- Wed, 17.05.2023, 16:45, CUBE 218
- Tue, 23.05.2023, 08:45, WZ105
- Wed, 24.05.2023, 08:45, CUBE 221 (tutorial)

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Part III: Macro Longevity Risk

1. Introduction
2. Relevance of Macro Longevity Risk
   - First Pillar: AOW
   - Second Pillar: Pension Funds
3. Modeling Mortality
4. Benchmark Model
   - The Lee-Carter Model
   - Alternative Estimation
   - Some Applications and Extensions
5. The AG2022 Model and COVID-19
   - Model and Projections
   - Closure of the Life Table
6. Model Risk: A Very Brief Introduction

Part IV: Pricing under all Types of Risk

7. Setting
8. Illustrations
   - No risk
   - Micro longevity risk
   - Macro longevity risk
   - Interest rate risk
   - All risks combined

LS + financial market risk

Part III

Macro Longevity Risk

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- **Micro Longevity Risk**

  Risk because (for given death probabilities) an individual's *remaining lifetime* is unknown.

  The remaining lifetime of an individual of age x belonging to a group g at time t is modeled as a random variable conditional on the future death probabilities $q^{(g)}_{x+s,t+s}$, $s = 0, 1, 2, \ldots$

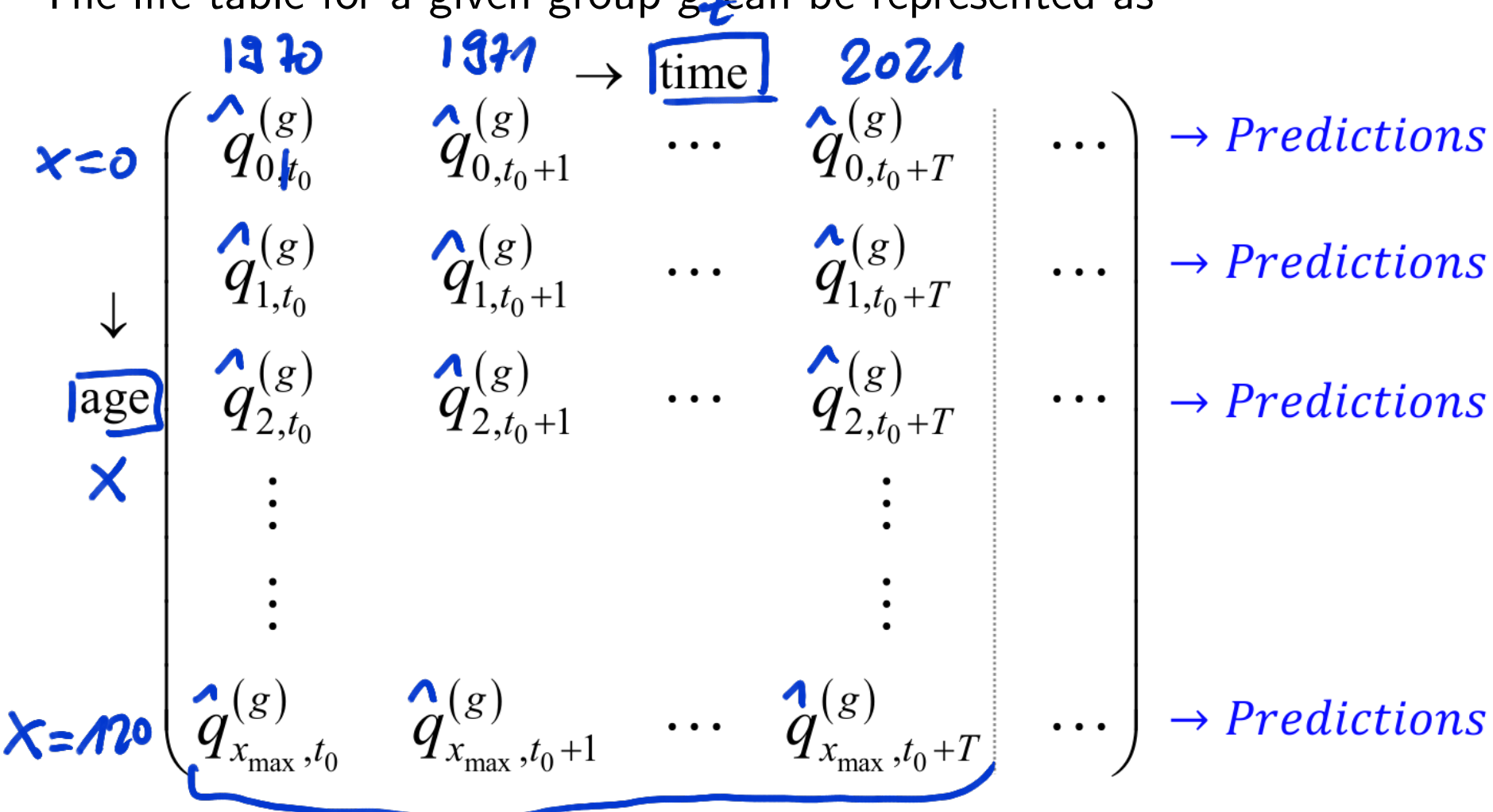
- **Macro Longevity Risk**

  Additional risk because future death probabilities are unknown.

  The future death probabilities $q^{(g)}_{x+s,t+s}$, $s = 0, 1, 2, \ldots$, will be modeled as random variables on date t.

Life Table of Group g

The life table for a given group g can be represented as



AG2022: $x_{\max=120}$, observed: $t_0 = 1970$, $t_0 + T = 2021$, predicted: $t_0 + T + s \geq 2022$

Life Table of Group g: 2 Questions

1. How to estimate/calibrate the observed part? → Data
2. How to determine the predictions and the uncertainty surrounding these predictions (macro longevity risk)? → Model

$$
\begin{array}{c}
 \\
\text{age} \downarrow
\end{array}
\begin{array}{c}
\rightarrow \text{time} \\
\left(\begin{array}{cccc|c}
\hat{q}^{(g)}_{0,t_0} & \hat{q}^{(g)}_{0,t_0+1} & \cdots & \hat{q}^{(g)}_{0,t_0+T} & \cdots \\
\hat{q}^{(g)}_{1,t_0} & \hat{q}^{(g)}_{1,t_0+1} & \cdots & \hat{q}^{(g)}_{1,t_0+T} & \cdots \\
\hat{q}^{(g)}_{2,t_0} & \hat{q}^{(g)}_{2,t_0+1} & \cdots & \hat{q}^{(g)}_{2,t_0+T} & \cdots \\
\vdots & & & \vdots & \\
\hat{q}^{(g)}_{x_{\max},t_0} & \hat{q}^{(g)}_{x_{\max},t_0+1} & \cdots & \hat{q}^{(g)}_{x_{\max},t_0+T} & \cdots
\end{array}\right)
\end{array}
\begin{array}{l}
 \\
\rightarrow \text{Predictions} \\
\rightarrow \text{Predictions} \\
\rightarrow \text{Predictions} \\
 \\
\rightarrow \text{Predictions}
\end{array}
$$

Period Calculations and Cohort Calculations

- **Period Calculations**
  - Period calculations is using the columns (e.g., copy the final column) of a life table to predict the next period death probability.
  - This means that any future changes to mortality rates would not be taken into account.
  - Period life expectancies use mortality rates from a single year and assume that those rates apply throughout the remainder of a person's life.

- **Cohort Calulations**
  - Cohort calculations is taking future trends into account using models.
  - A cohort life table uses a combination of observed mortality rates for the cohort for past years and projections about mortality rates for the cohort for future years.
  - Requires a model.

(x, t)

$\rightarrow$ Period life expectancy would match cohort life expectancy only if there were no changes in age-specific mortality rates over time.

Traditional Approach: Naive Forecast

- Traditionally, macro longevity risk was ignored.
- One assumed that the most recently estimated period death probabilities hold true for all future years, i.e., for the cohort $(x, t_0 + T)$ one assumed

$$\underbrace{q^{(g)}_{x+s,t_0+T+s}}_{\textit{predicted}} = \underbrace{q^{(g)}_{x+s,t_0+T}}_{\textit{observed}}$$

  for all $s \geq 0$ and all ages x.
- This means that – if we ignore macro longevity risk – the entries in the last column of the observation part of the life table equal the entries of the prediction part.

Traditional Approach: Naive Forecast

$$q_{x+s,\,t+s} = q_{x+s,\,t} \qquad t = t_0 + T$$

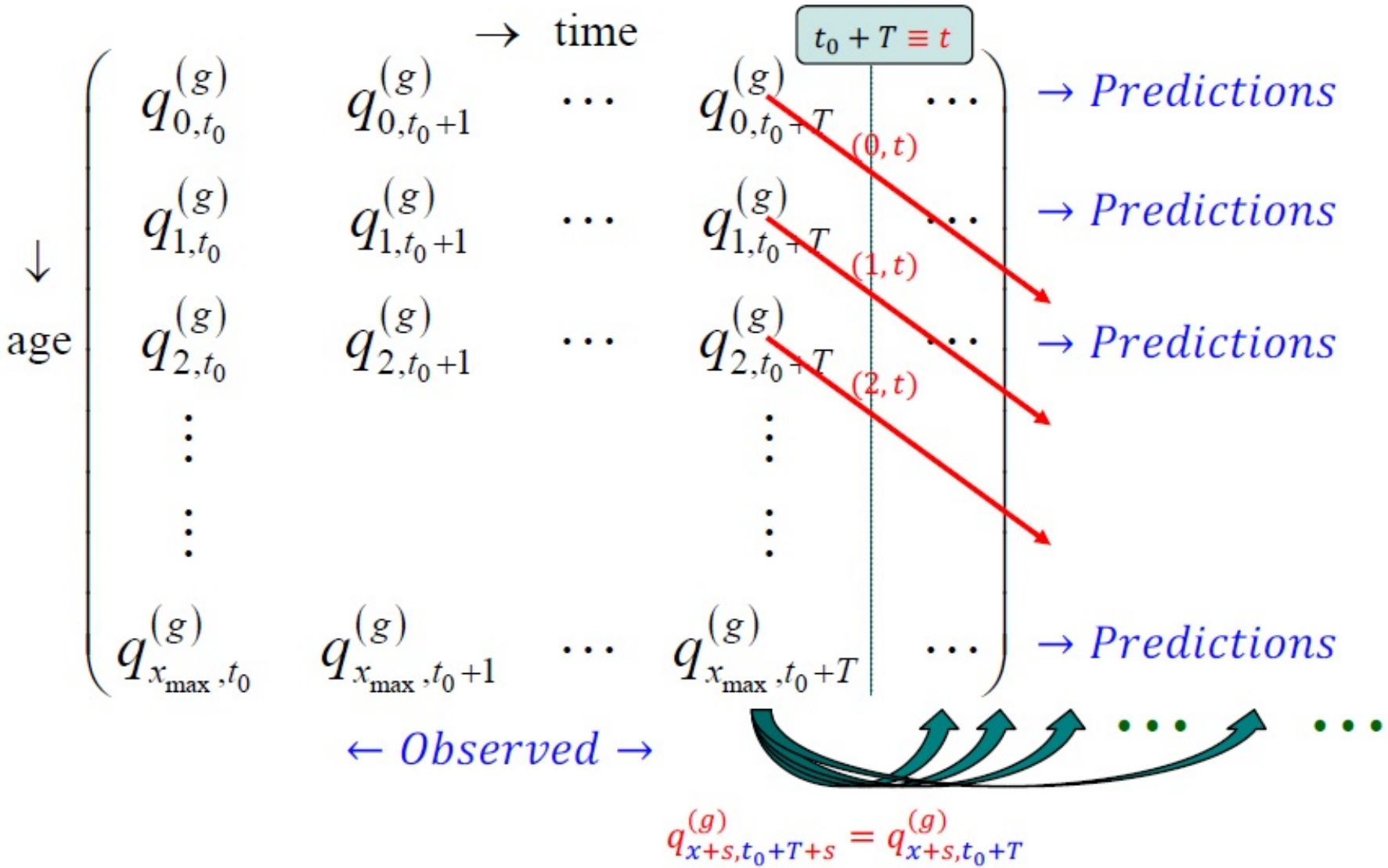
$s=0$: $q_{x,t} = q_{x,t}$ tautological

$s=1$: $\boxed{q_{x+1,t+1}} = q_{x+1,t}$

$s=2$: $q_{x+2,t+2} = q_{x+2,t}$

Cohort calculations and period calculations yield the same output.

Traditional Approach: Naive Forecast



Period vs. Cohort Calculations

Both period and cohort calculations have some drawbacks:

- **Drawbacks of period calculations**
  - Ignoring trends in death probabilities may lead to significant overestimation of death probabilities.
  - Ignoring uncertainty in future death probabilities may lead to significant underestimation of the risk in life insurance portfolios.
  - Sensitive to (transitory) shocks, e.g., WW2, Spanish flu, COVID-19.
- **Drawbacks of cohort calculations**
  - We unavoidably introduce model risk if we use forecasts.

Current Practice

- Statistics Netherlands (CBS) and the Royal Dutch Actuarial Association (AG) produce point forecasts for future one-year death probabilities by age and gender.

  $\rightarrow$ Are available on the website of the AG.

- These point forecasts are referred to as *best-estimate death probabilities*.

- The AG-models also easily allow the quantification of (at least part of the) macro longevity risk.

- To mitigate the effect of model risk, these best-estimate death probability forecasts are revised annually (CBS, December), or bi-annually (AG, September in even years).

Recall: Some Formulas for Cohort (x, t)

- **τ-years-from-now survival probability:**

$$ {}_{\tau}p_{x,t}^{(g)} = \prod_{k=0}^{\tau-1} p_{x+k,t+k}^{(g)}, \qquad p_{x,t}^{(g)} = 1 - q_{x,t}^{(g)} $$

- **Remaining life expectancy:**

$$ e_{x,t}^{(g)} = \sum_{\tau=1}^{\infty} {}_{\tau}p_{x,t}^{(g)} + 0.5 $$

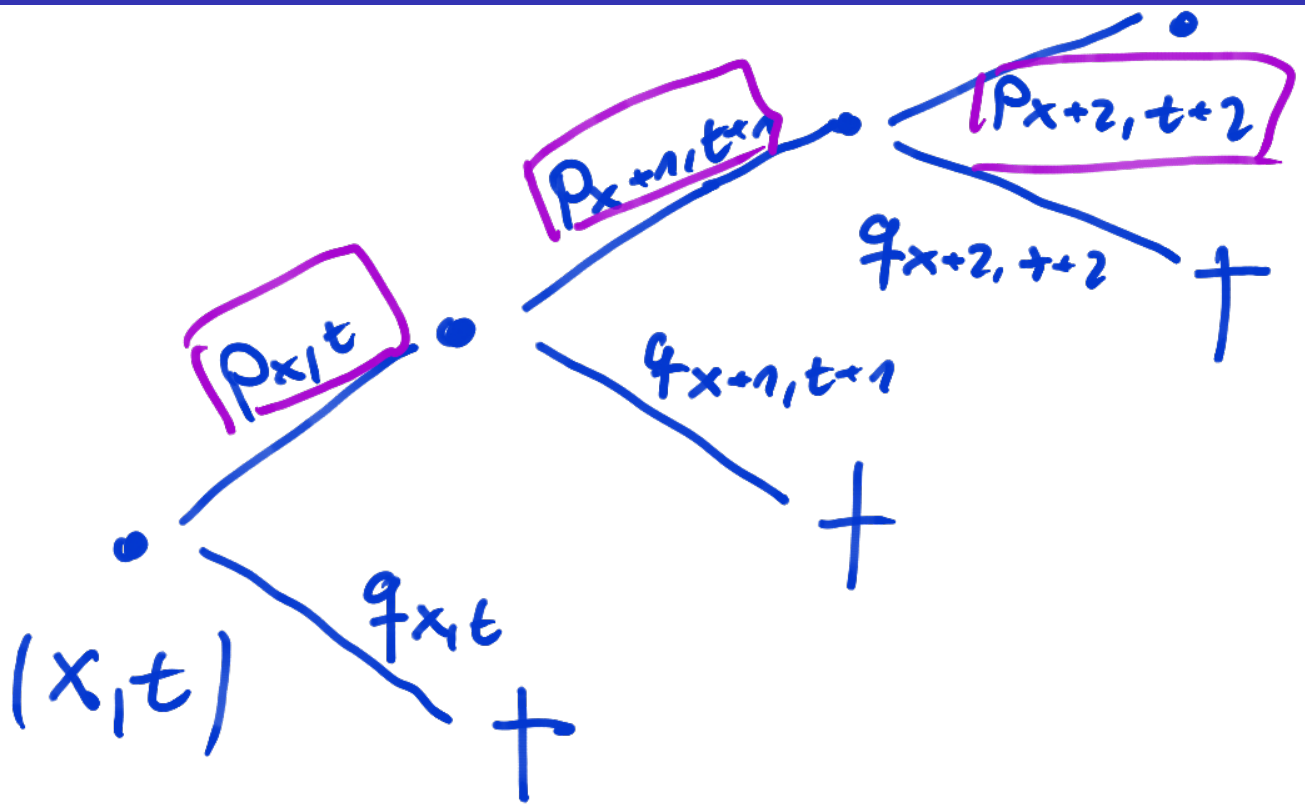
- **Value of immediate single life annuity:**

$$ a_{x,t}^{(g)} = \sum_{\tau=1}^{\infty} {}_{\tau}p_{x,t}^{(g)} \frac{1}{\big(1 + R_t(t+\tau)\big)^{\tau}} $$

- **Value of T-years deferred single life annuity:**

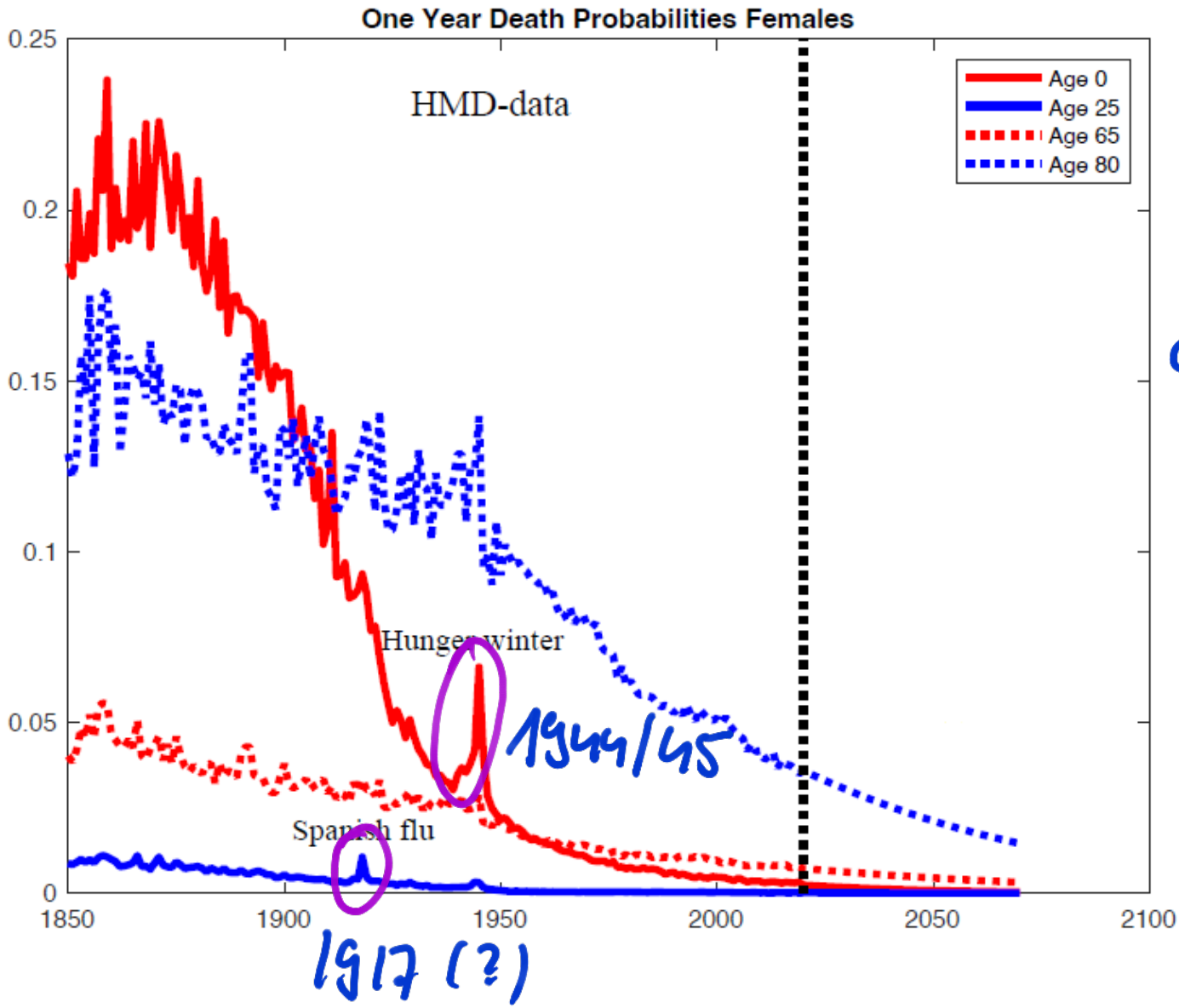
$$ a_{x,t}^{(g)}(T) = \sum_{\tau=T}^{\infty} {}_{\tau}p_{x,t}^{(g)} \frac{1}{\big(1 + R_t(t+\tau)\big)^{\tau}} $$

Recall: Some Formulas for Cohort (x, t)



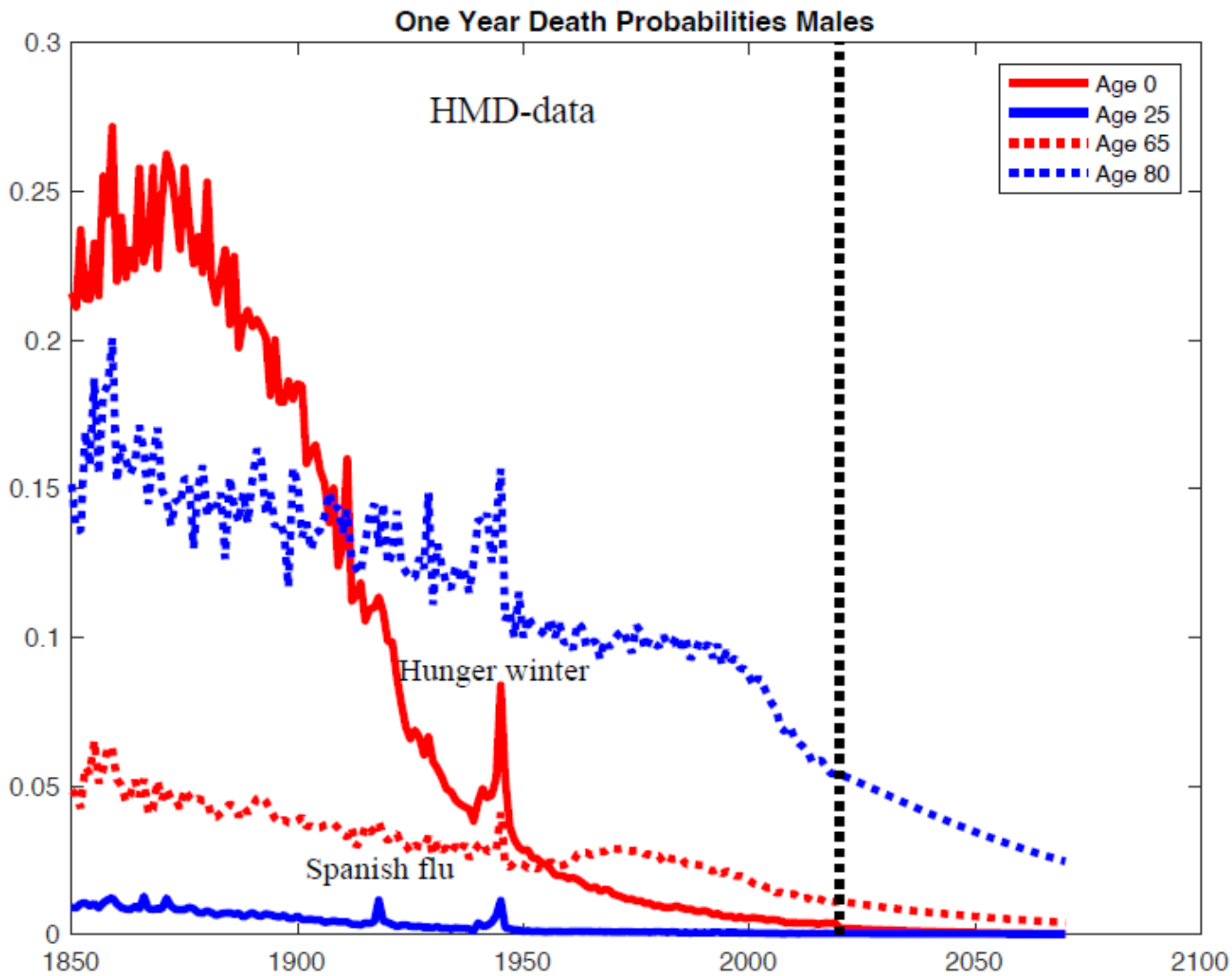
$$P(T_{x,t} \geq 3) = \prod_{s=0}^{2} p_{x+s,t+s}$$

One-Year Death Probabilities

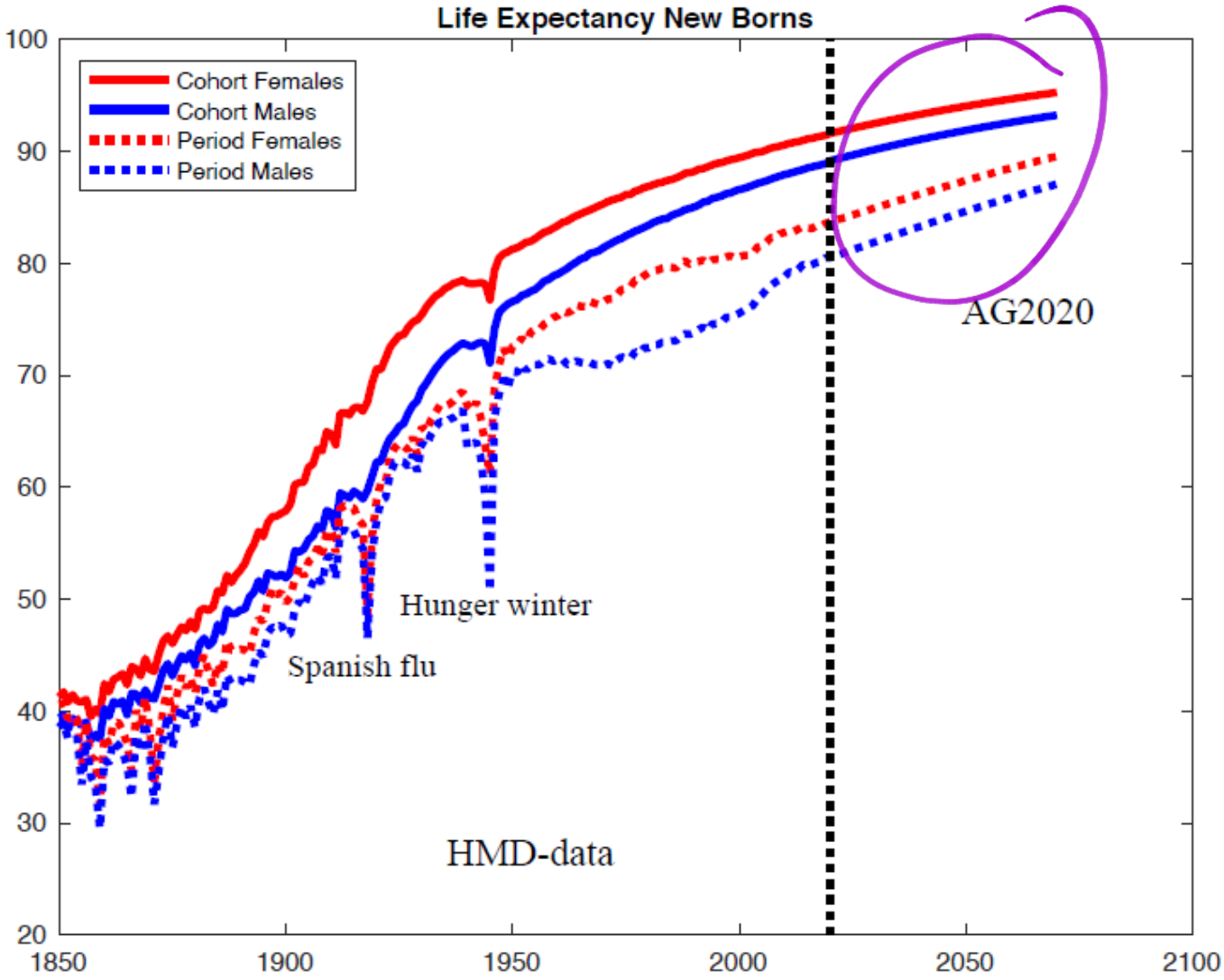


death probabilities have declined dramatically

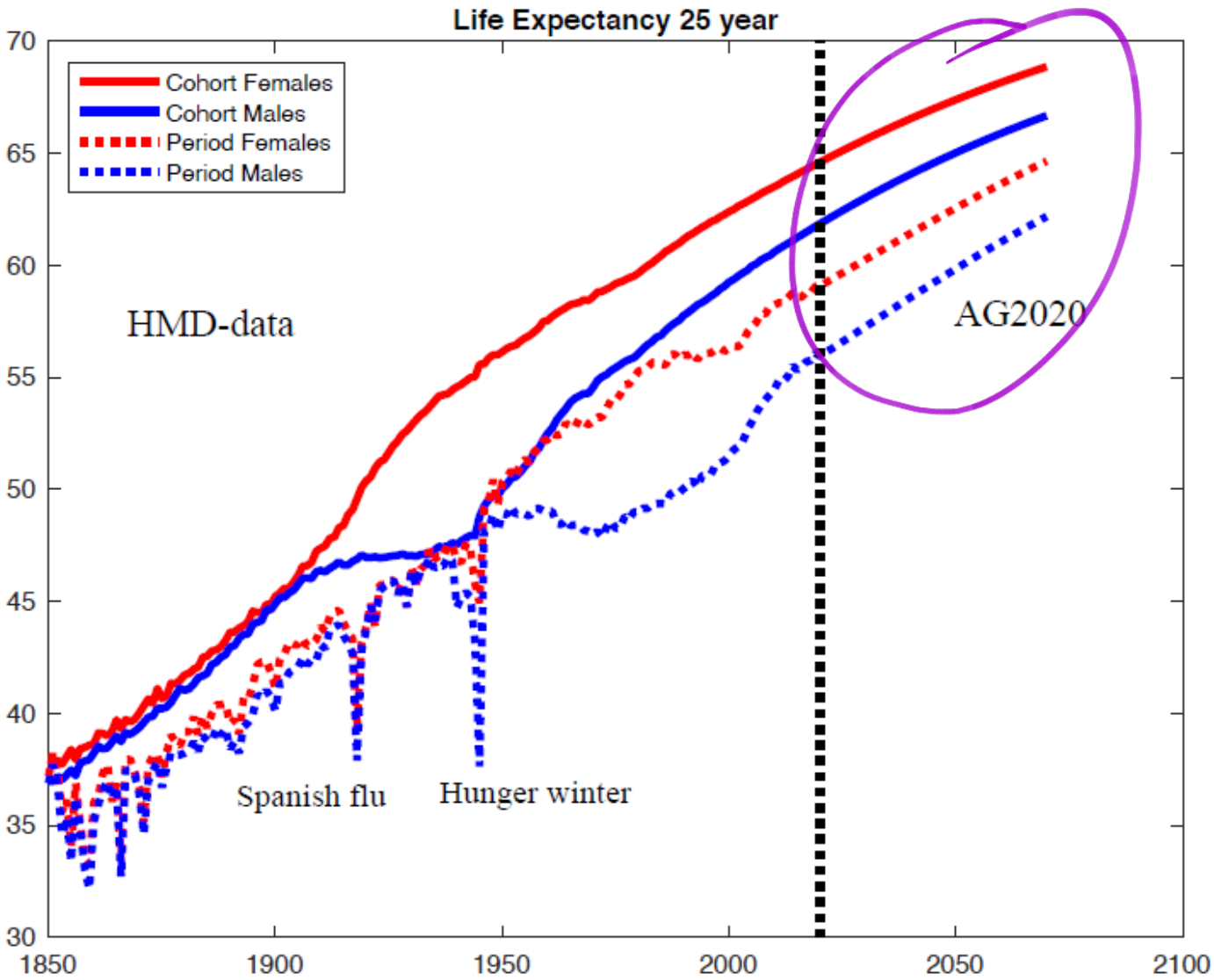
One-Year Death Probabilities



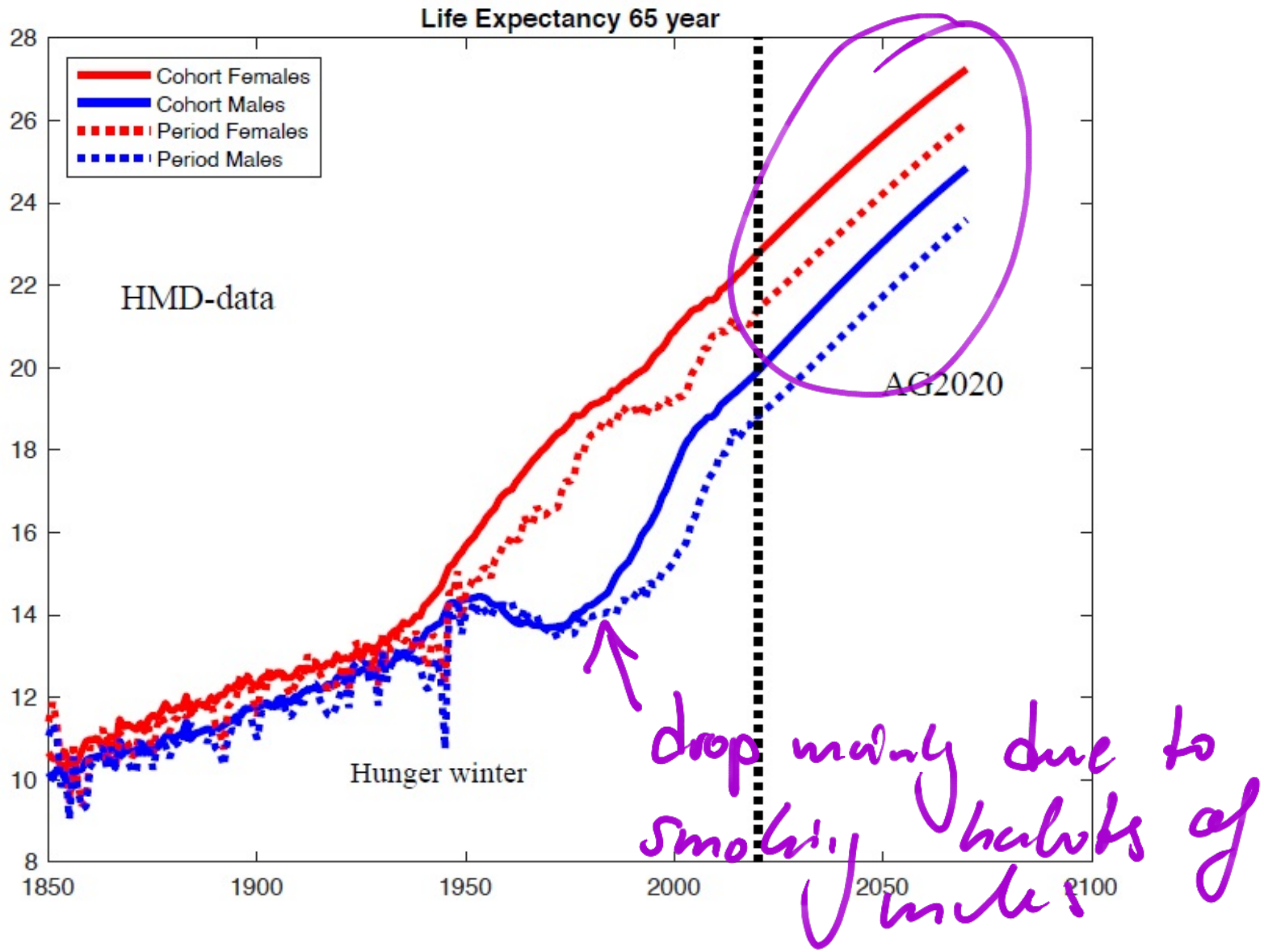
Remaining Life Expectancy (Newborns)



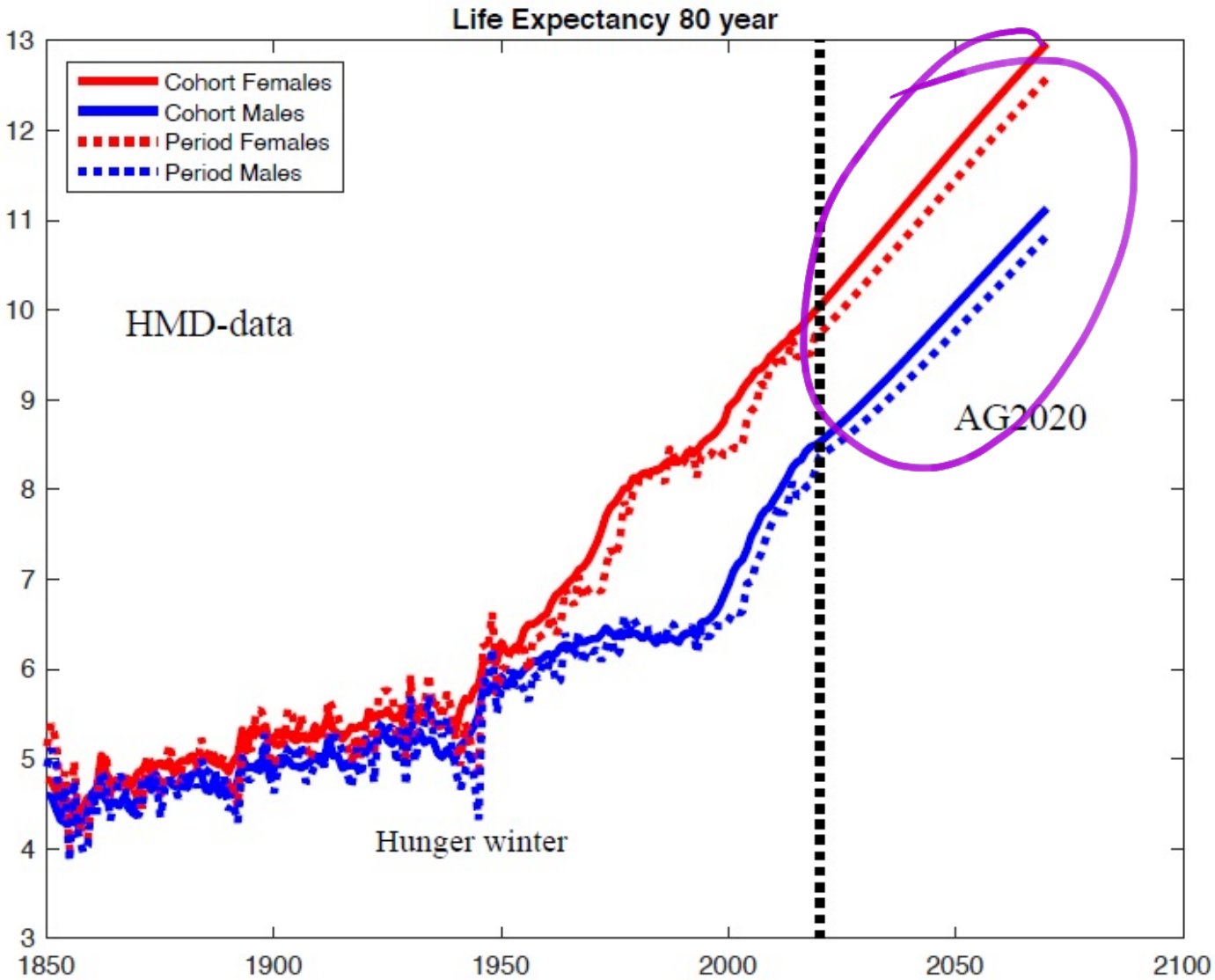
Remaining Life Expectancy (Age 25)



Remaining Life Expectancy (Age 65)



Remaining Life Expectancy (Age 80)



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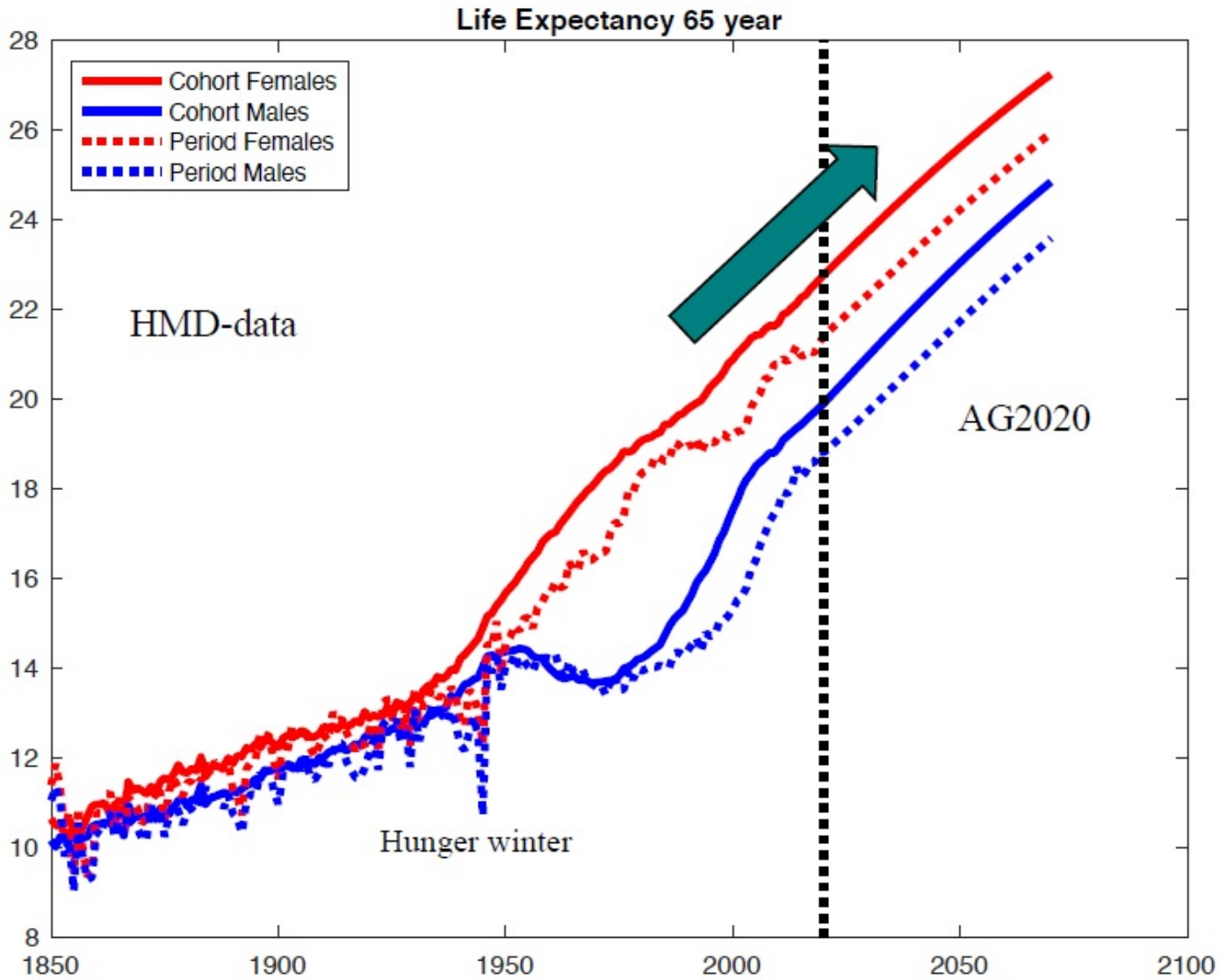
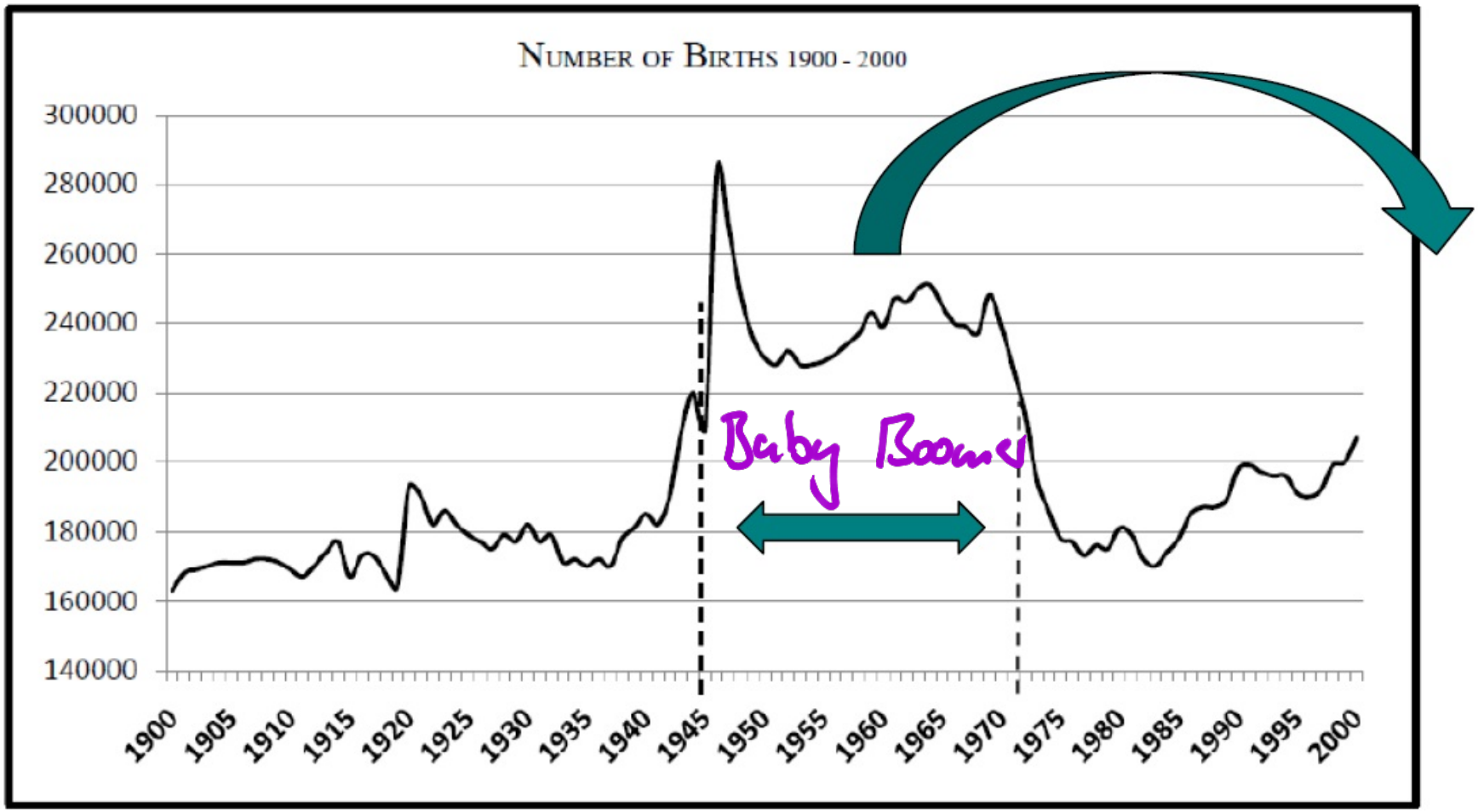
First Pillar: AOW

- AOW is the basic pension in the Netherlands that everyone gets, who lived in the Netherlands.
- This pillar is not related to how much the retiree worked.
- The pension depends on how many years the retiree lived in the Netherlands before retirement.
- If the retiree lived fifty years before retirement in the Netherlands, he/she gets the full amount. If someone lived a shorter period of time in the Netherlands, this amount will be scaled down proportionally.
- Changes in life expectancy can affect whether the government can afford AOW.
  - Life expectancy has increased dramatically during the last decades.
  - It is unclear whether and how it will continue to increase (macro longevity risk).
- Other factors such as the number of newborns influence the stability and sustainability of the pension system.

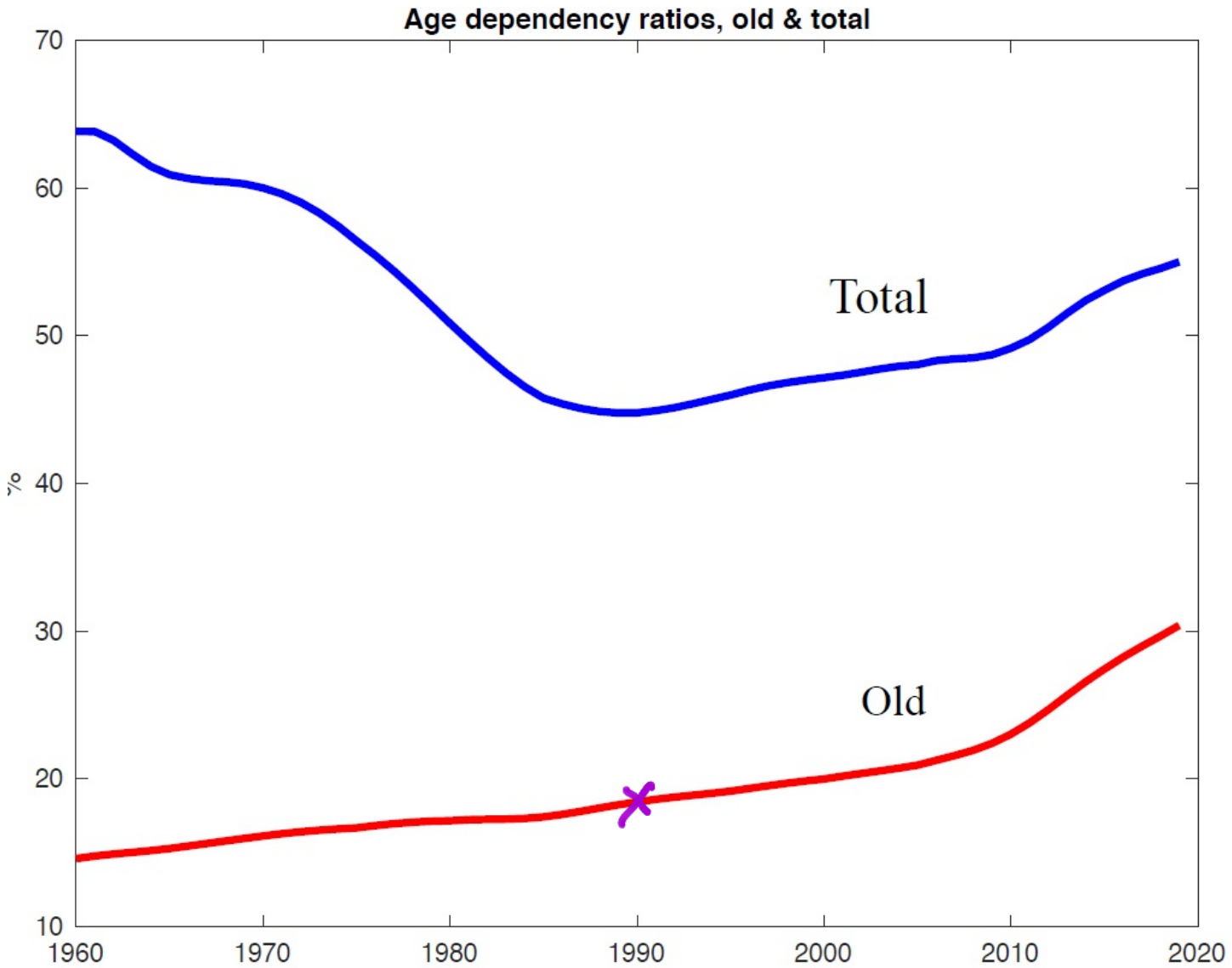
Increase in Life Expectancy



Number of Newborns (Source: CBS)



Dependency Ratio (Source: World Bank)



Young & Old / Working

Old / Working

Possible Future Scenarios (Source: CBS)

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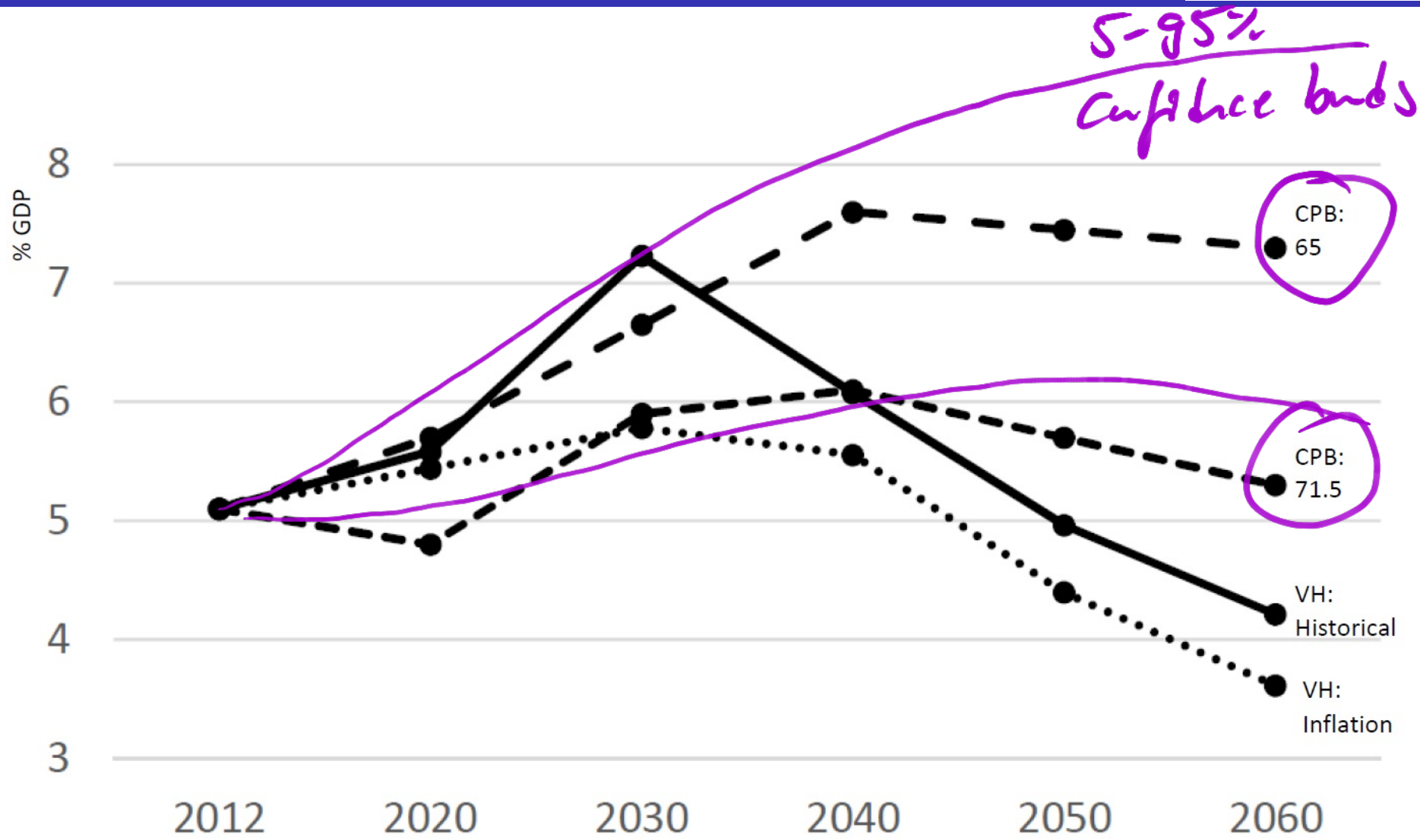
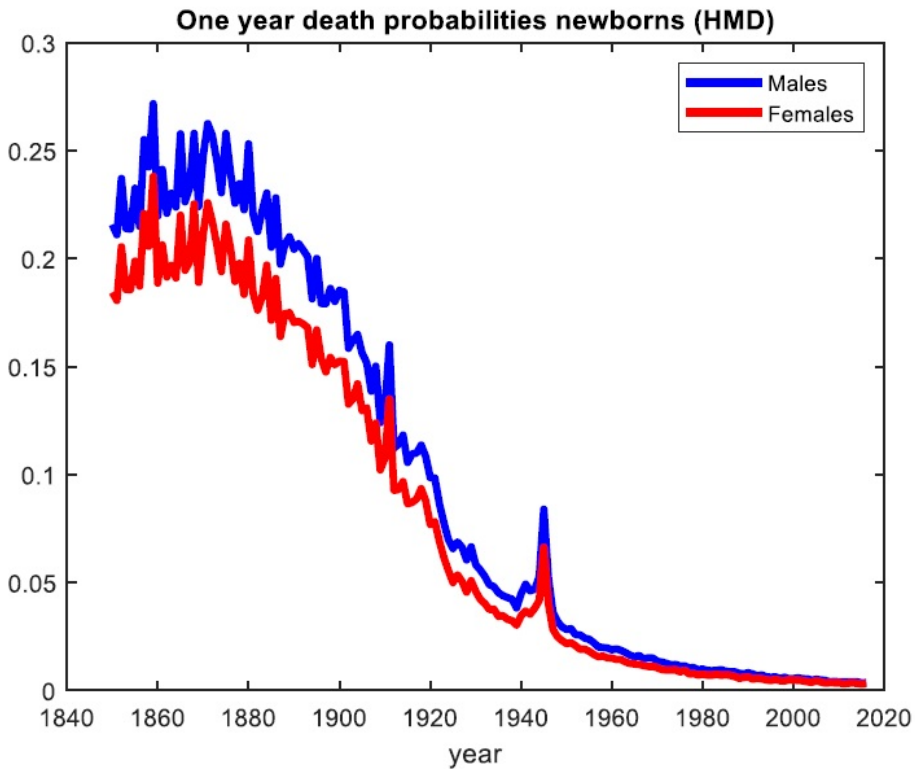
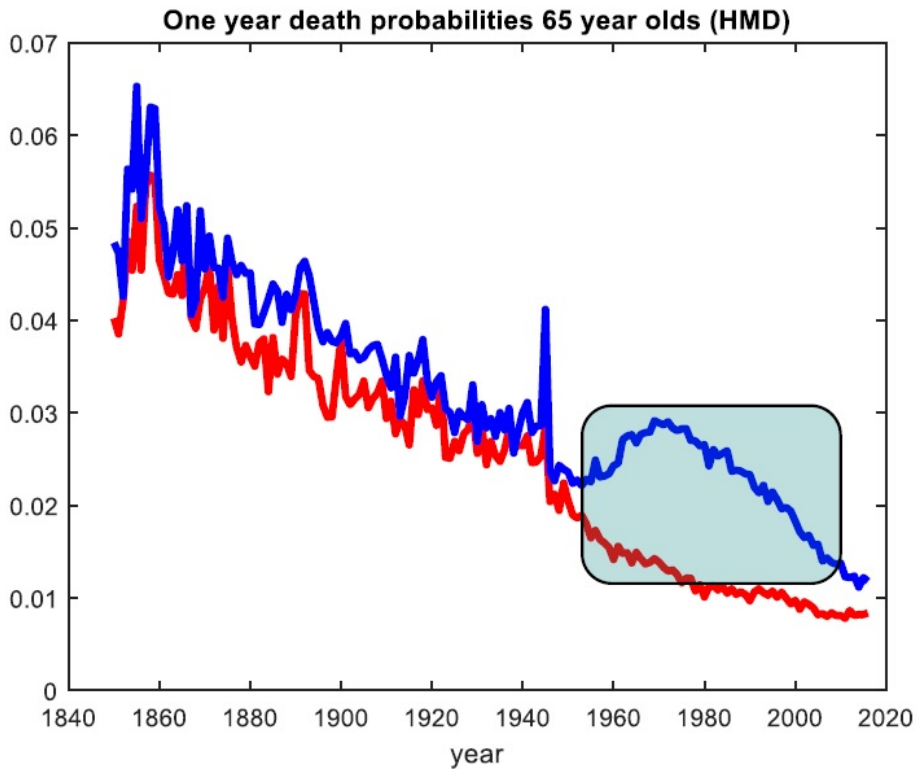
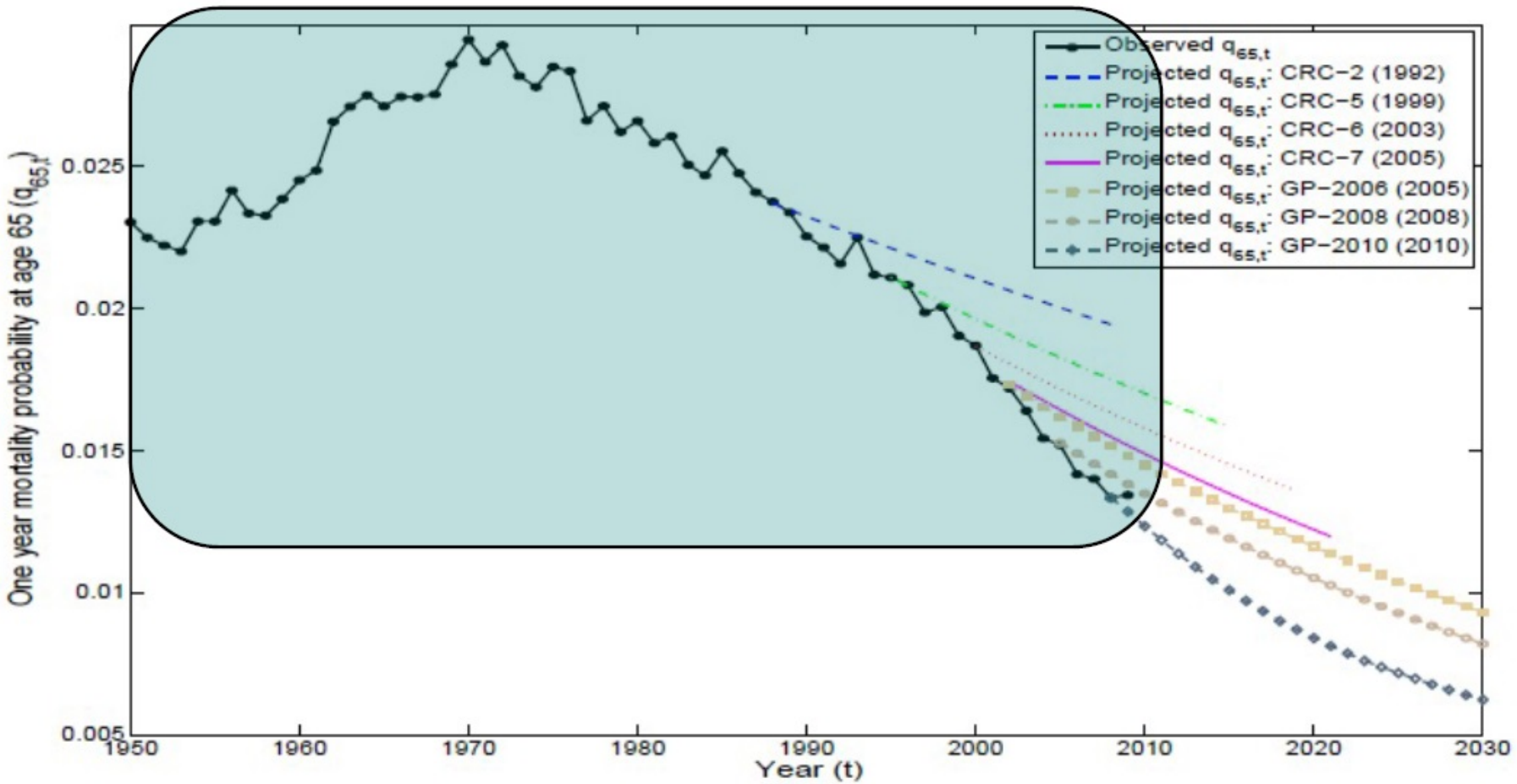


Possible Future Scenarios





Possible Future Scenarios



- Best estimate projections were wrong in the past!
- Macro Longevity Risk: Need to quantify the uncertainty around the projections as well.

Current Practice Revisited

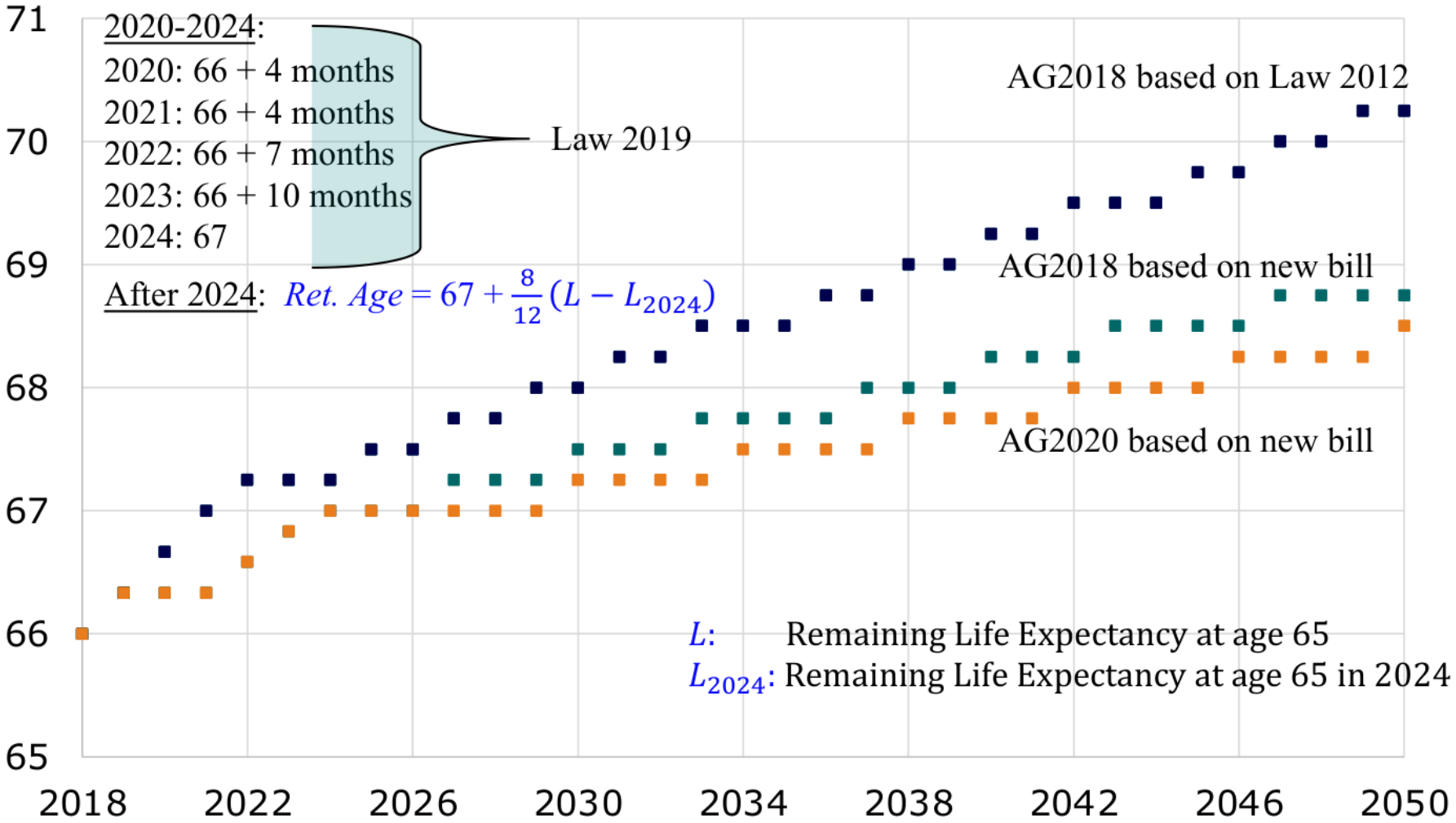
- Statistics Netherlands (CBS) and the Royal Dutch Actuarial Association produce point forecasts for future one-year death probabilities by age and gender.

  $\rightarrow$ Are available on the website of the AG.

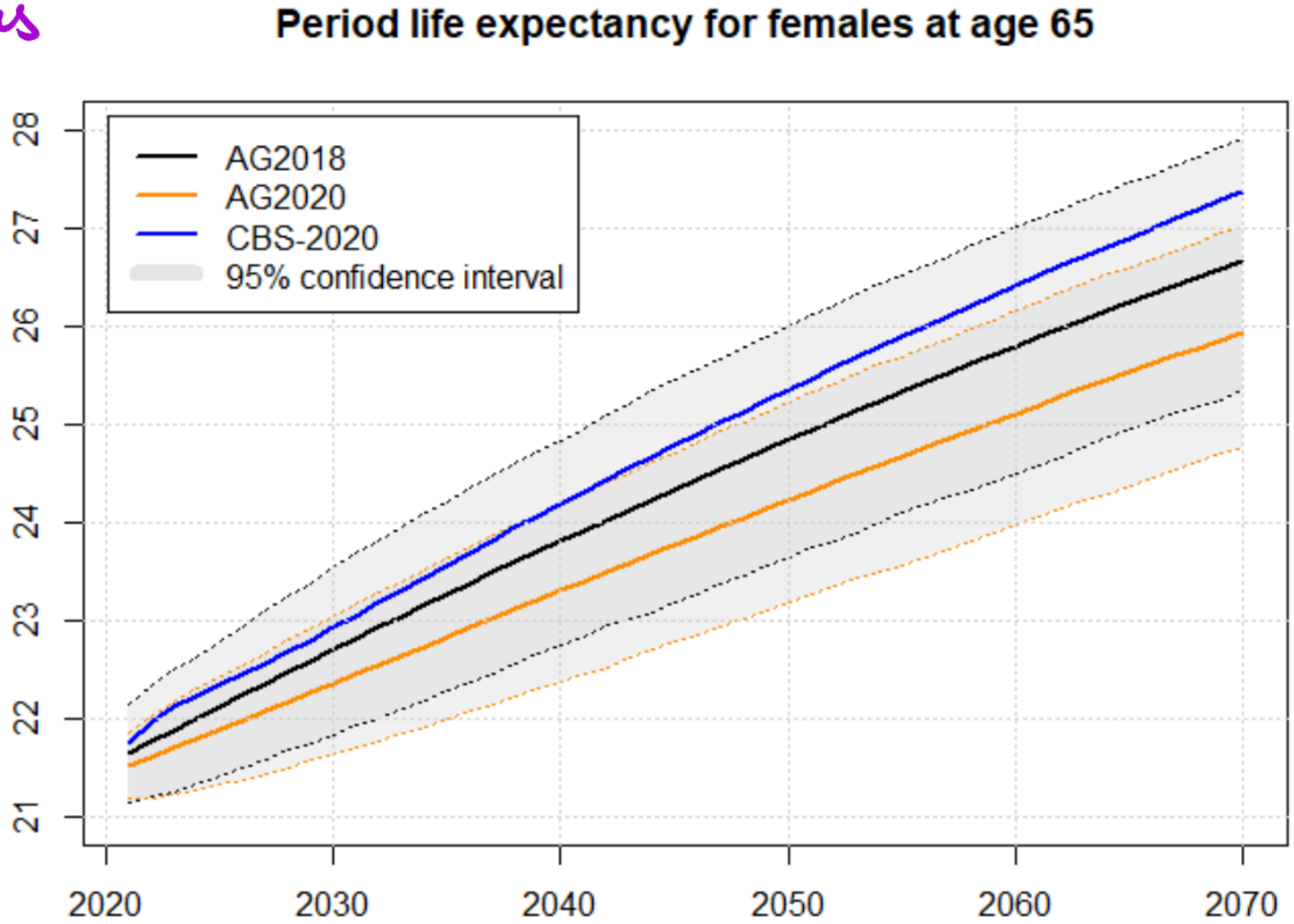
- These point forecasts ("best-estimate" death probabilities) are nowadays based on underlying models. These models can also be used to quantify macro longevity risk, for example, in terms of confidence intervals around the point forecasts.

- Part III of the course is going to illustrate this.
  - The models are not only used to derive the best estimates.
  - They can also be used to estimate confidence intervals describing the uncertainty around the point estimates.

Illustrating Macro Longevity Risk



Females


Period life expectancy for females at age 65
AG2018
AG2020
CBS-2020
95% confidence interval
21
22
23
24
25
26
27
28
2020
2030
2040
2050
2060
2070

Illustrating Macro Longevity Risk

Males