

**Capital Markets and Asset Pricing**  
**Goethe Business School**  
**Summer Term 2022**  
**Dr. Christoph Hambel**  
**Final Exam**

- The exam is open book, i.e., any dead objects may be used.
- Any kind of communication with other persons will be considered cheating.
- The working time is 90 minutes.
- The exam consists of three problems. The maximum number of points in every problem is 30, so that the maximum number of total points in the exam is 90.
- There are two types of problems:
  1. Problems with a text entry field

You are supposed to answer those questions by writing a text.

2. Problems with a number input box

**Input box in LPlus for xx**

These boxes will be corrected automatically in LPlus. If your answer is correct, you will get the points. You will also have the opportunity, but not the obligation, to explain your calculations at the end of each problem. I will check your explanations only if your solution in the box is wrong. In this case, I can give you some points if your documentation is understandable and evident.

**Problem 1 (Bond Pricing)** There are four bonds traded in the market, two riskless Treasury bonds, and two risky corporate bonds. The first Treasury bond has a maturity of one year and an annual coupon of 1%, and a second Treasury bond which has a maturity of two years and an annual coupon of 2%. The first corporate bond has a maturity of one year and an annual coupon of 4%, and the second corporate bond has a maturity of two years and an annual coupon of 8%. All bonds have a notional of 100 euros. The Treasury bonds are currently traded at par while the corporate bonds are traded at 95 euros.

- (a) Calculate the continuously compounded zero rates for the Treasury bonds. What are the zero rates of the corporate bonds? (4 credits)

**Input box in LPlus for  $r_T(1)$ : 1%**

**Input box in LPlus for  $r_T(2)$ : 1.99%**

**Input box in LPlus for  $r_C(1)$ : 9.05%**

**Input box in LPlus for  $r_C(2)$ : 10.41%**

- (b) What are the durations and convexities of the Treasury bonds? (*HINT: The yield-to-maturity of the two-year Treasury bond is 1.98%.*) (8 credits)

**Input box in LPlus for  $D_1$ : 1**

**Input box in LPlus for  $D_2$ : 1.98**

**Input box in LPlus for  $\Gamma_1$ : 1**

**Input box in LPlus for  $\Gamma_2$ : 3.94**

- (c) Explain how one would use the duration to approximate the price change given a yield shift. What can be a problem? How can this problem be resolved? (4 credits)

(d) Comment on the following statements: (6 credits)

- (i) A higher duration leads to higher bond price volatility.
- (ii) The duration increases with higher coupons.

(e) Assume that the recovery rate on the corporate bonds is 60% of the notional and the coupon is lost if the firm defaults. What is the risk-neutral survival probability of the company over the first year? What is the expected loss? (4 credits)

**Input box in LPlus for  $SP^Q$ : 81.70%**

**Input box in LPlus for  $EL^Q$ : 7.32**

(f) The government has recently decided to emit a consol bond with an annual coupon payment of 50 euros. The consol bond is being traded at a price of 1750 euros. Determine its discretely compounded and continuously compounded yield-to-maturity, respectively. (4 credits)

**Input box in LPlus for  $y_d$ : 2.86%**

**Input box in LPlus for  $y_c$ : 2.82%**

**Problem 2 (State Pricing and Credit Risk)** Assume that there are three traded securities in the market:

- a risk-free treasury zero-coupon bond with a notional of 1 dollar and a current price of 99 cents
- a risky corporate zero-coupon bond with a notional of 1 dollar and a current price of 96 cents
- stocks with a current price of 100 dollars

The stocks and corporate bonds are issued by the same firm. In one year, there are three possible scenarios (boom, stagnation, recession). In the boom and stagnation scenario, the company does not default and repays the bond. The stock price is assumed to be 120 dollars in the boom scenario and 80 dollars in the stagnation scenario. In the recession scenario, the company defaults and 50% of the notional of the corporate bond is recovered.

- (a) Determine the discretely compounded risk-free rate in this economy. *(2 credits)*

**Input box in LPlus for  $r$ : 1.01%**

- (b) Elaborate on the stock price in the recession scenario. *(2 credits)*

- (c) Determine the state prices of the three states. *(6 credits)*

**Input box in LPlus for  $\pi_b$ : 0.64**

**Input box in LPlus for  $\pi_s$ : 0.29**

**Input box in LPlus for  $\pi_r$ : 0.06**

If you are unable to determine the state prices, you may assume  $\pi_b = 0.58$ ,  $\pi_s = 0.35$ ,  $\pi_r = 0.06$  from here on.

- (d) Elaborate on the existence or non-existence of arbitrage opportunities and market completeness in this setting. (4 credits)

The firm plans to issue junior debt in form of a zero-coupon bond with a notional of 1 dollar. Assume that in the boom and stagnation scenario the firm is able to fully repay this bond, but will default on this bond in the third scenario. The recovery rate is assumed to be zero.

- (e) What is the fair price of this bond at time zero? (4 credits)

**Input box in LPlus for  $P_0$ : 0.93**

- (f) Calculate the discretely compounded yield and the spread of the junior bond. (4 credits)

**Input box in LPlus for  $y_{junior}$ : 7.53%**

**Input box in LPlus for  $s_{junior}$ : 6.52%**

- (g) Assume that you hold the junior bond. Name two derivatives that can eliminate the default risk from your position in the junior bond. Pick one of these derivatives, write down its payoff structure and determine its arbitrage-free price. (8 credits)



### Problem 3 (Capital Asset Pricing Model)

(a) Are the following statements true or false? Explain your answers carefully.

- (i) In the CAPM world, all investors hold a portfolio where all idiosyncratic risk is diversified away. *(3 credits)*

- (ii) An asset whose expected return is below the security market line is overvalued. *(3 credits)*

- (iii) A put option written on the market portfolio has positive market beta. *(3 credits)*

(b) Describe in one short paragraph the differences between CAPM and APT. *(5 credits)*

(c) Assume that the risk-free rate is 0.5% and the expected market return is  $\mu_M = 6\%$ . The volatility of market return is estimated to be 20%. An exchange-traded fund has a beta of 0.75 with respect to the market portfolio and a volatility of 25%.

(i) Determine the systematic risk and the idiosyncratic risk of this fund. (5 credits)

**Input box in LPlus for systematic risk: 0,0225**

**Input box in LPlus for idiosyncratic risk: 0,04**

(ii) Calculate the correlation between the fund and the market (4 credits)

**Input box in LPlus for  $\rho_{M,P}$ : 0.6**

(iii) How large is the correlation between the fund and the risk-free asset? (2 credits)

**Input box in LPlus for  $\rho_{M,r}$ : 0**

(d) Explain why Roll's critique does not apply to the Arbitrage Pricing Theory. (5 credits)