Part IV

Pricing under all Types of Risk

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Ultimate Goal of the Lecture

- Now, we have gathered all the tools and techniques to price life insurance contracts and annuities in a realistic setting.
- We consider three different types of risk
  - Micro Longevity Risk (Part I): risk because (for given death probabilities) an individual's *remaining lifetime* is unknown.
  - Interest Rate Risk (Part II): risk because future *interest rates* are unknown.
  - Macro Longevity Risk (Part III): risk because *future death probabilities* are unknown.
- We study the pricing of life insurance contracts and annuities in various settings:
  - No risk (tedious but useful)
  - Micro longevity risk
  - Macro longevity risk
  - Interest rate risk
  - All risks combined

Recall: Three Sources of Risk

1. Micro Longevity Risk (i.i.d.): $N^{(g)}_{x+t,T+t} \sim \mathcal{B}(N^{(g)}_{x,T},\ {}_t p^{(g)}_{x,T})$

$$N^{(g)}_{x+t,T+t} = \underbrace{N^{(g)}_{x,T}\ {}_t p^{(g)}_{x,T}}_{\text{best estimate}} + \underbrace{E^{(g)}_{x+t,T+t}}_{\text{forecast error}}$$

2. Macro Longevity Risk (Lee-Carter): $p^{(g)}_{x,T+t} = \mathrm{e}^{-m^{(g)}_{x,T+t}}$,
$\ln(m^{(g)}_{x,t}) = \alpha^{(g)}_x + \beta^{(g)}_x \kappa^{(g)}_t + \varepsilon^{(g)}_{x,t}$

$$\kappa^{(g)}_{T+t} = \underbrace{\kappa^{(g)}_T + t \cdot c^{(g)}}_{\text{best estimate}} + \underbrace{\delta^{(g)}_{T+1} + \cdots + \delta^{(g)}_{T+t}}_{\text{forecast error}}$$

3. Interest Rate Risk (Vasicek): $r_{t+1} = \mu + \theta r_t + \sigma \varepsilon_{t+1}$, $R_t(t+1) = r_t$

$$r_{T+t} = \underbrace{\mu \sum_{i=1}^{t-1} \theta^i + \theta^t r_T}_{\text{best estimate}} + \underbrace{\sigma \sum_{i=1}^{t} \theta^{t-i} \varepsilon_{T+i}}_{\text{forecast error}}$$

Recall: Three Sources of Risk

Setting

- We only consider a single group g (g suppressed from now on) at time t.
- Members of this group belong to a cohort (x, t). The number of individuals belonging to cohort (x, t) is given by $N_{x,t}$.
- All individuals have bought an immediate single life annuity from a fund at time t. This annuity promises to pay off 1 unit per period, starting in the next period $t + 1$.

  $\rightarrow$ This assumption is not realistic but helps us come up with closed-form solutions to illustrate the impact of the various risk sources.
- The fund invests the received payments in assets, in order to be able to pay off the promised amounts of the annuities.

Setting: Annuity Prices

- The annuity prices are based on "best estimate" cohort life table at time t, explicitly indicated by $BE(t)$, i.e.,

$$a_{x,t}^{BE(t)} = \sum_{\tau=1}^{\infty} {}_{\tau}p_{x,t}^{BE(t)} \frac{1}{\big(1 + R_t(t+\tau)\big)^{\tau}}.$$

- The pension fund's total liabilities $L_t^{BE(t)}$ are given by

$$L_t^{BE(t)} = \sum_{x \in \mathcal{X}} N_{x,t} a_{x,t}^{BE(t)}.$$

- At time t the fund's total assets are denoted by A_t.

Recall: Funding Ratio

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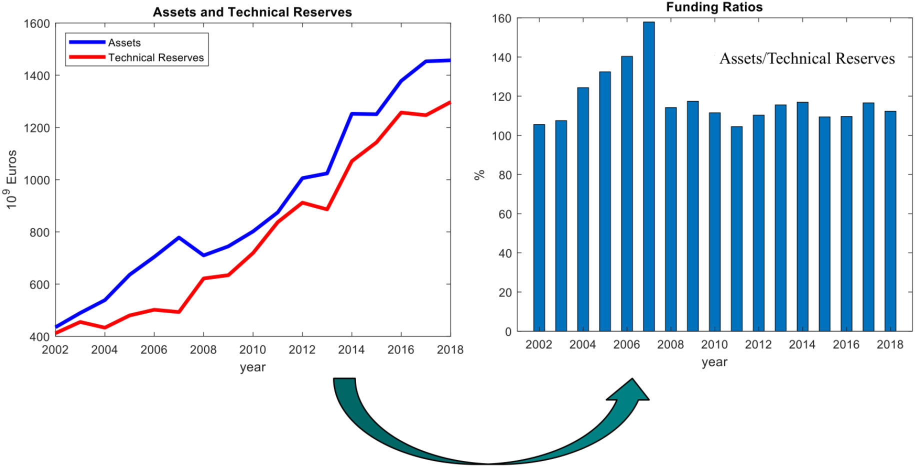

Setting: Funding Ratio

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- Thus, at time t, the fund's *funding ratio* (FR_t) is defined as

$$FR_t^{BE(t)} = \frac{A_t}{L_t^{BE(t)}}.$$

- The funding ratio does obviously depend on all three sources of risk under consideration.
- We ask the question: What can we say about FR_{t+1}, considering
  - No risk
  - Micro longevity risk
  - Macro longevity risk
  - Interest rate risk
  - All risks combined