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- So far, we have dealt with the Lee & Carter (1992) model and some of its variants. We now look into the AG2022 Model and COVID-19.
- The AG2022 Model consists of three layers (a so-called three-layer Lee-Li model):
    - A layer for the European population.
    - A correction layer for the Dutch population.
    - An extra layer for COVID-19.
- The first two layers are as in AG2020.

$$\ln\left(\mu_{x,t}^{g}\right) = A_x^g + B_x^g K_t^g + \alpha_x^g + \beta_x^g \kappa_t^g + \mathfrak{B}_x^g\, \mathfrak{K}_t^g$$

- A_x^g: Fixed Age effect Europe
- B_x^g: Dynamic Age effect Europe
- K_t^g: Time effect Europe
- α_x^g: Fixed Age effect Deviation NL
- β_x^g: Dynamic Age effect Deviation NL
- κ_t^g: Time effect Deviation NL
- $\mathfrak{B}_x^g$: Dynamic Age effect COVID in NL
- $\mathfrak{K}_t^g$: Time effect COVID in NL

Relation to AG2020: First Two Layers

- The AG2022 directly builds upon AG2020.
- Surrounding countries selected based upon comparable welfare level.
- AG2020 re-estimated with data EU 2019 added (but data 2020 & 2021 not included).
  - EU data: $D_{x,t}^{(g),EU}$, $E_{x,t}^{(g),EU}$, $t = 1970, \dots, 2019$, $x = 0, \dots, 90$
  - Dutch data: $D_{x,t}^{(g),NL}$, $E_{x,t}^{(g),NL}$, $t = 1970, \dots, 2019$, $x = 0, \dots, 90$
- AG2020 is basis for long-term projections.
- The third layer is due to COVID-19 and is new. It is calibrated using data from 2020 and 2021.

AG2020 Model Re-estimated, $t \leq 2019$

- Number of deaths ($D_{x,t}^{(g)}$) modeled assuming a Poisson-distribution with expectation equal to exposure ($E_{x,t}^{(g)}$) times force of mortality ($\mu_{x,t}^{(g),pre-covid}$):

$$D_{x,t}^{(g)} \mid E_{x,t}^{(g)} \sim \mathcal{P}\left(E_{x,t}^{(g)} \mu_{x,t}^{(g),pre-covid}\right)$$

- The long-term mortality trend is estimated based on the European reference group:

$$\ln \mu_{x,t}^{(g),pre-covid,EU} = A_x^{(g)} + B_x^{(g)} K_t^{(g)}$$

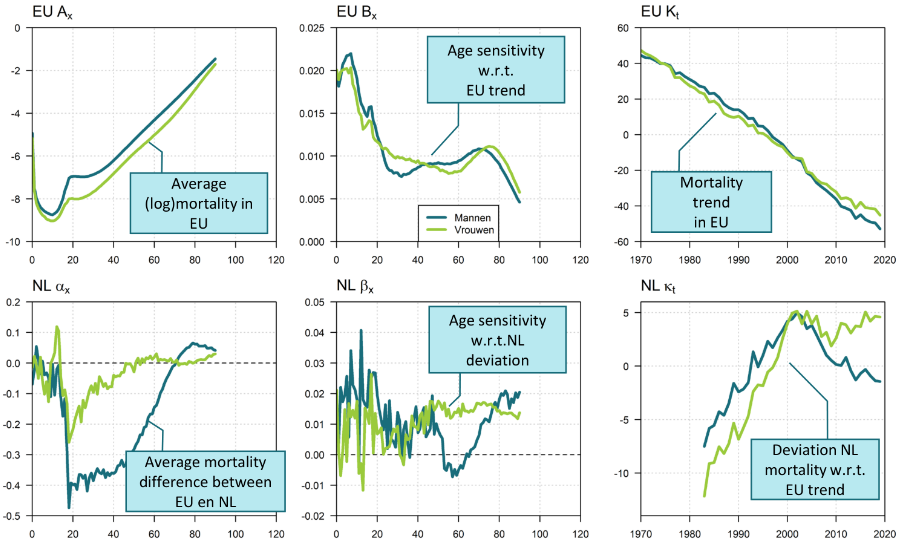
- For the Netherlands mortality is estimated conditional on the force of mortality of the European reference group:

$$\ln \mu_{x,t}^{(g),pre-covid} = A_x^{(g)} + B_x^{(g)} K_t^{(g)} + \alpha_x^{(g)} + \beta_x^{(g)} \kappa_t^{(g)}$$

- All parameters are estimated by maximizing the corresponding likelihood functions.

AG2020 Model Re-estimated, $t \leq 2019$

Estimation


EU A_x
Average (log)mortality in EU
EU B_x
Age sensitivity w.r.t. EU trend
Mannen
Vrouwen
EU K_t
Mortality trend in EU
NL α_x
Average mortality difference between EU en NL
NL β_x
Age sensitivity w.r.t.NL deviation
NL κ_t
Deviation NL mortality w.r.t. EU trend

Intuition

- The European period effects, $K_t^{(g)}$, are assumed to follow a random walk with drift (as in Lee-Carter):

$$\Delta K_t^{(g)} = \theta^{(g)} + \varepsilon_t^{(g)}$$

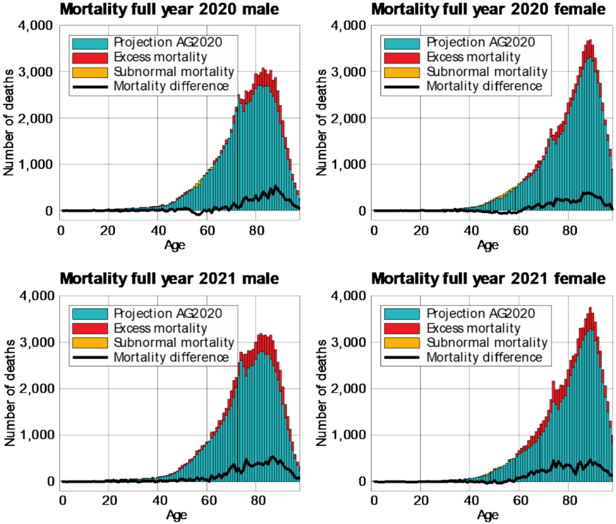
- The Dutch period effects, $\kappa_t^{(g)}$, are assumed to follow a first-order autoregressive process with constant:

$$\Delta \kappa_t^{(g)} = a^{(g)} \kappa_{t-1}^{(g)} - c^{(g)} + \delta_t^{(g)}$$

- The error terms are assumed to follow a multivariate normal distribution with mean vector 0 and covariance matrix C.
- This combination of assumptions results in coherent projections:
  - In the long run the difference between the Dutch and the European death probabilities converges to zero, but:
  - In the short run the Dutch and European death probabilities might deviate.

AR(1)-Process

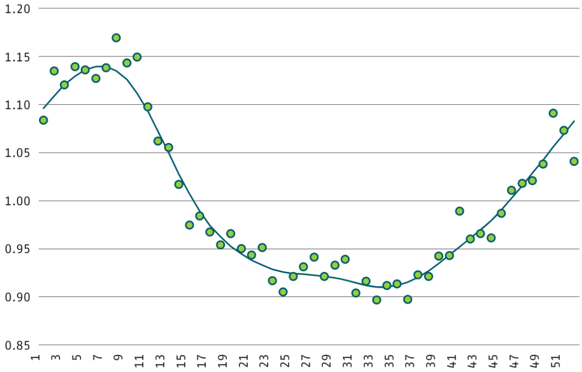
Third Layer: COVID-19



- Clear impacts of COVID-19 in both 2020 and 2021.
- Age effect in AG2022 differs from age effect in AG2020.

Third Layer: COVID-19

- Data from 2020 and 2021 cannot be used for normal update.
- Change to weekly data:
  - Customized data needed, plus interpolation and extrapolation.
  - Seasonal effect.
  - New age- and time effect.



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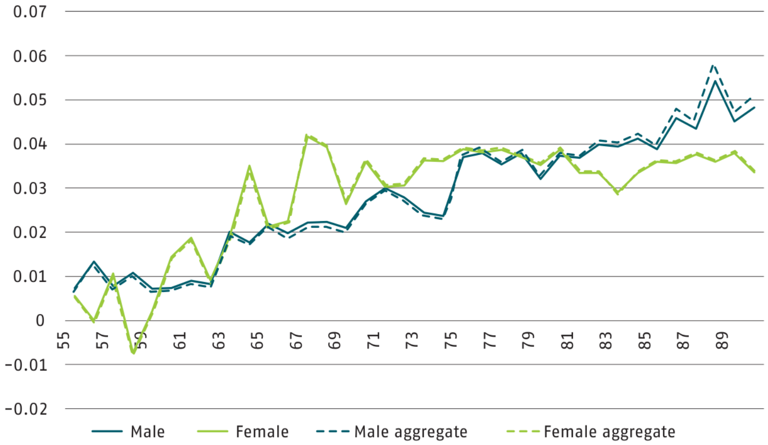
- To describe the deviation in mortality in 2020 and 2021 relative to what is expected on the basis of data before 2020, we introduce a new age effect $\mathfrak{B}_x^{(g)}$ and new (week-based) timeseries $\mathfrak{K}_{w,2020}^{(g)}$ for 2020 and $\mathfrak{K}_{w,2021}^{(g)}$ for 2021.
- The seasonal effect determined earlier is also added.
- Three factors:
  - the pre-pandemic estimate for a given year (an AG2020 model update with datapoints until 2019),
  - a seasonal effect for a given week,
  - a new factor representing the impact of the altered circumstances since the pandemic:

$$\ln \mu_{x,t}^{(g)} = \ln \mu_{x,t}^{(g),pre-covid} + \ln \phi_{w,t} + \mathfrak{B}_x^{(g)} \mathfrak{K}_{w,t}^{(g)}$$

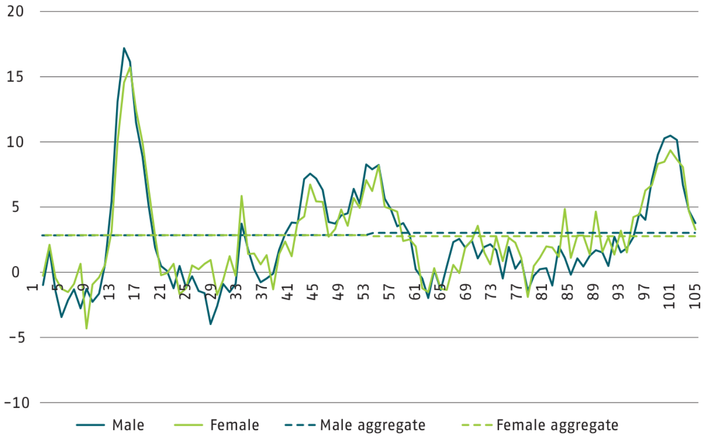
- The time effect of COVID-19 $\mathfrak{K}_{w,t}^{(g)}$ varies significantly across weeks.

Third Layer: Age Effect



- Using weekly data allows the estimation of age effects based on more than 2 (annual) data points.
- No effect until age 55, constant from age 90.

Third Layer: Time Effect



- Time series for difference in mortality (whether or not due to COVID) relative to AG2020 model (with update 2019 data).
- Aggregating week effects $\mathfrak{K}_{w,t}^{(g)}$ gives estimate of the impact $\mathfrak{X}_t^{(g)}$ for the whole year (dashed lines).

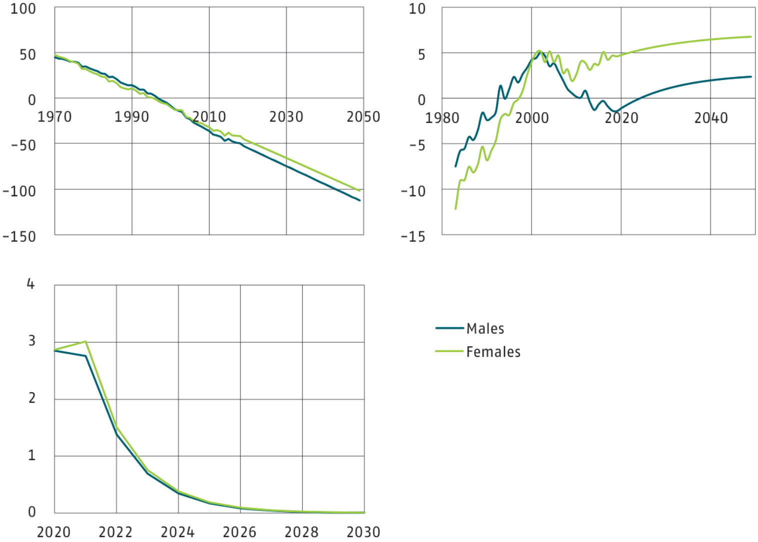
Third Layer: Projection Model

- Estimating the impact for a whole year is the starting point for projecting future years.
- For the course of the projection further assumptions are needed.
- Choice CSO: disappearing (exponentially), i.e.,

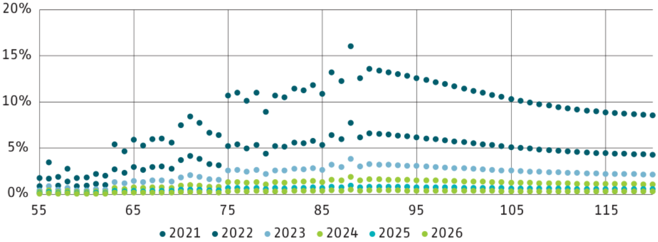
$$\mathfrak{X}_t^{(g)} = \mathfrak{X}_{2021}^{(g)} \eta^{t-2021}, \qquad t \geq 2021$$

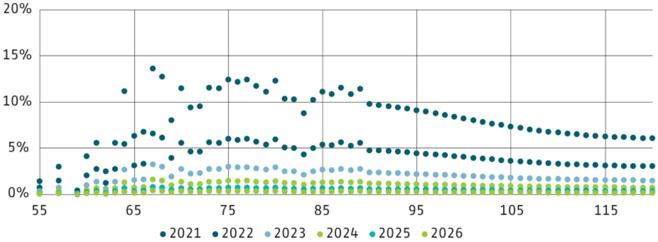
  with $\eta = 1/2$, which implies that the half-life of the impact equals one year.
- This assumption determines the 'best estimates' for all future values of the time series in the model.

Third Layer: Projection Model



Third Layer: Projection Model



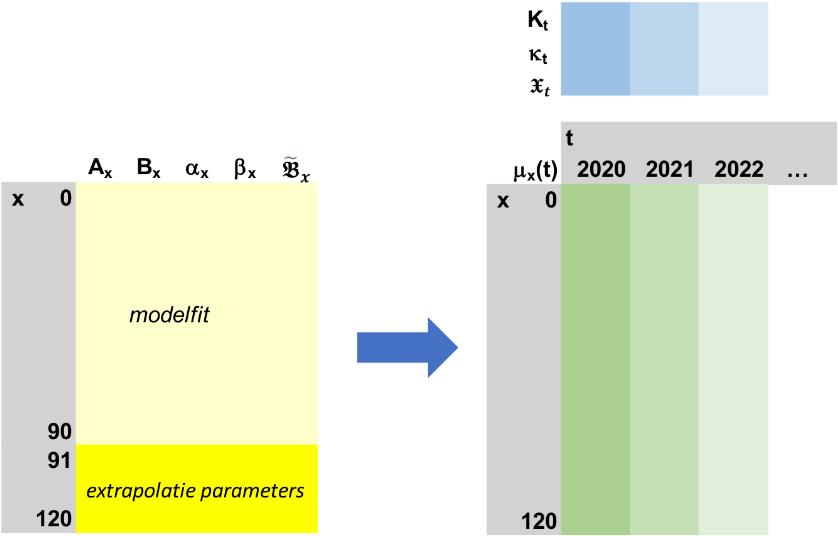


- *Problem:* The AG2022 model uses mortality data for $x = 0, \ldots, 90$. If $x \geq 91$, the data becomes rather noisy, because the population older than 90 is small.
- To deal with this issue, the AG2020 model used a popular method to extrapolate mortality rates for older ages – the *Kannisto closing method*.
- For the ages $x \in \{91, \ldots, 120\}$, $\widehat{\mu}^{(g)}_{x,T+t}$ is determined as

$$\widehat{\mu}^{(g)}_{x,T+t} = L\Big(\sum_{k=80}^{90} w_k(x) L^{-1}(\widehat{\mu}^{(g)}_{k,T+t})\Big),$$

where $L(z) = \frac{1}{1+\mathrm{e}^{-z}}$ and $L^{-1}(z) = \ln\left(\frac{z}{1-z}\right)$ and some weight function w_k (see Assingment).

Closure of the Life Table



Issues with Kannisto

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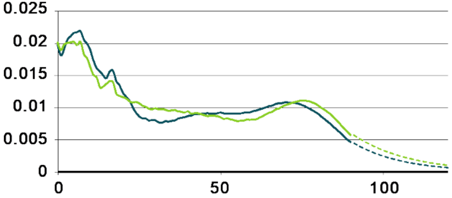
- The Kannisto method was standard in the AG-models and has been standard in many other forecasting models to deal with sparse data points in older cohorts.
- This method has some drawbacks:
  - Not all ages show improved mortality over time. For higher ages mortality rises monotonously to a positive and limit value that is known with certainty, $\lim_{t\to\infty} \widehat{q}^{(m)}_{x,T+t} = 0.6321$, $x \geq 101$. As a result, life expectancy also converges to a limit value known with certainty $\lim_{t\to\infty} e^{(m)}_{0,t} = 102.08$.
  - Because the limit values are known with certainty, uncertainty decreases (smaller confidence intervals) over time, while we expect increasing uncertainty (wider confidence intervals).
- The new method in AG2022 addresses this issue and closes the life table without knowing limit values with certainty.

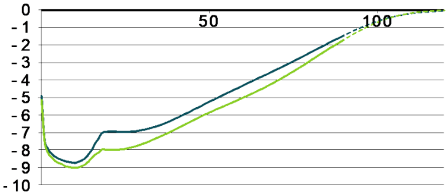
Issues with Kannisto

B_x^g



- Extrapolate $\ln(B_x^g)$ linearly
- Interpretation:
  all ages benefit from decreasing trend in K_t

A_x^g

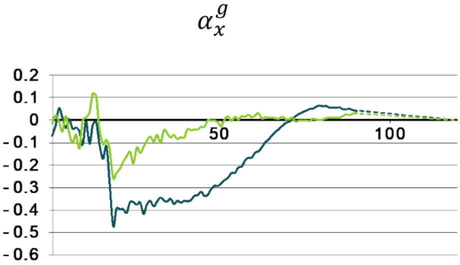


- Determine A_x^g such that in the last sample period (2019) the same EU death probabilities result as in case of Kannisto

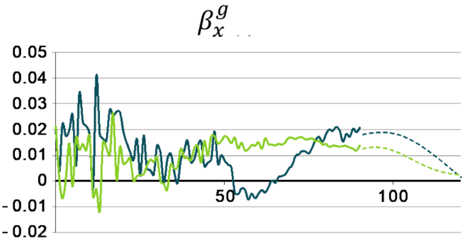
male ------ extrapolation

female ------ extrapolation

Closure of the Life Table



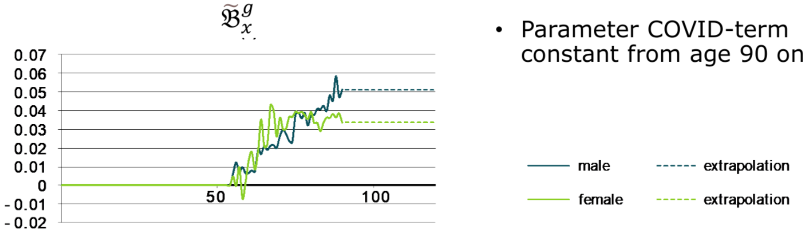
- Extrapolate α_x^g linearly to $\alpha_{120}^g = 0$
- Interpretation:
  for age 120 no difference between NL and EU.

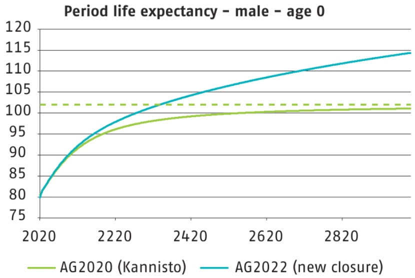


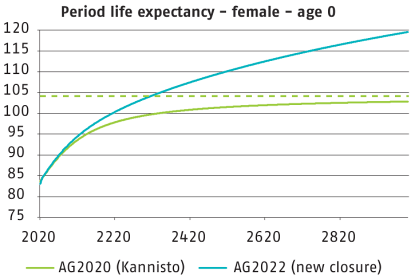
- Determine β_x^g such that in the last sample period (2019) the same NL death probabilities result as in case of Kannisto.

Closure of the Life Table

- Parameter COVID-term constant from age 90 on







- The new closing method does not have the undesirable characteristics, which we observe in the application of Kannisto per projection year.
- There is no turning point at a certain age: also, at higher ages the death probabilities keep decreasing.
- The life expectancy does not have a limit whose value is known in advance.
- The confidence intervals do not become smaller over time.