APPENDIX TO
Life Insurance Demand under Health Shock Risk

This version: December 14, 2015

Christoph Hambel

Goethe University, Faculty of Economics and Business Administration,

Frankfurt am Main, Germany,

email: christoph.hambel@finance.uni-frankfurt.de

Holger Kraft

Goethe University, Faculty of Economics and Business Administration,

Frankfurt am Main, Germany,

email: holgerkraft@finance.uni-frankfurt.de

Lorenz S. Schendel

Goethe University, Faculty of Economics and Business Administration,

Frankfurt am Main, Germany,

email: schendel@safe.uni-frankfurt.de

Mogens Steffensen

University of Copenhagen, Department of Mathematical Sciences,

Copenhagen, Denmark,

email: mogens@math.ku.dk

A Optimization Problem

The family optimizes expected utility from intermediate consumption and terminal wealth. The optimization problem is characterized by several state variables: financial wealth x, labor income y, the health status of the wage earner H and the current insurance choice I. The control variables are the consumption rate c, the proportion of wealth θ invested in risky assets, and the impulse control strategy for the insurance decision (ζ_i, ω_i), $i \in \mathbb{N}$. At time $t = 0$ the wage earner is assumed to be 20 years old. The optimization problem is then given by

$$\max_{\{c_s,\theta_s\}_{s\in[0,T)},\{(\zeta_i,\omega_i)\}_{i\in\mathbb{N}}} \mathrm{E}_{0,x,y,I,H}\left[\int_0^T \mathrm{e}^{-\delta u}\,\frac{\left(\frac{c_u}{\phi_{H_u}}\right)^{1-\gamma}}{1-\gamma}\,du + \varepsilon\,\mathrm{e}^{-\delta T}\,\frac{{X_T}^{1-\gamma}}{1-\gamma}\right] \tag{A.1}$$

$$\text{s.t.} \qquad dX_t = X_t\left[(r+\theta_t\lambda\sigma_S)\,dt + \theta_t\sigma_S\,dW_t^S\right] + \left[Y_t - c_t - \mathbb{1}_{\{A_t=1,2\,\wedge\,t<T_I\}}\iota(I_t)\right]dt \tag{A.2}$$

$$\qquad + \mathbb{1}_{\{H_{t^-}=1,2\,\wedge\,t<T_I\}} I_{t^-}\,dN_t^D,$$

$$X_{\zeta_i} = X_{\zeta_i^-} - \eta(\zeta_i,\omega_i,I_{\zeta_i}), \tag{A.3}$$

where we impose short-sale constraints, i.e. $\theta_t \in [0,1]$, and consider admissible strategies, which in particular requires $X_t \geq 0$.[1] The value function (indirect utility function) is defined by

$$J(t,x,y,I,H) = \sup_{\{c_s,\theta_s\}_{s\in[t,T)},\{(\zeta_i,\omega_i)\}_{i\in\mathbb{N}}} \mathrm{E}_{t,x,y,I,H}\left[\int_t^T \mathrm{e}^{-\delta(u-t)}\,\frac{\left(\frac{c_u}{\phi_{H_u}}\right)^{1-\gamma}}{1-\gamma}\,du + \varepsilon\,\mathrm{e}^{-\delta(T-t)}\,\frac{{X_T}^{1-\gamma}}{1-\gamma}\right]. \tag{A.4}$$

To solve the problem, we split it into its impulse control and stochastic control part.

Given no intervention at t, but optimal impulse control afterwards, the value function is denoted

[1] Since income is not bounded away from zero and the family has a bequest motive, the latter condition is automatically satisfied.

by

$$J^*(t,x,y,I,H) = \sup_{\{c_s,\theta_s\}_{s\in[t,T)},\{(\zeta_i\,|\,\zeta_i\neq t,\omega_i)\}_{i\in\mathbb{N}}} \mathrm{E}_{t,x,y,I,H}\left[\int_t^T \mathrm{e}^{-\delta(u-t)}\,\frac{\left(\frac{c_u}{\phi_{H_u}}\right)^{1-\gamma}}{1-\gamma}\,du + \varepsilon\,\mathrm{e}^{-\delta(T-t)}\,\frac{{X_T}^{1-\gamma}}{1-\gamma}\right]. \tag{A.5}$$

In this case, the optimization problem reduces to a stochastic control problem. The corresponding Hamilton-Jacobi-Bellman equation (HJB) is given by

$$\begin{aligned}
\delta J^* = \sup_{c,\theta} \quad & \Bigg\{ \frac{\left(\frac{c}{\phi_H}\right)^{1-\gamma}}{1-\gamma} + J^*_t + J^*_x\left[x\left(r+\theta\lambda\sigma_S\right) + y - c - \mathbb{1}_{\{H=1,2\,\wedge\, t<T_I\}}\iota(I)\right] \\
& + \frac{1}{2}J^*_{xx}x^2\theta^2\sigma_S^2 + \mathbb{1}_{\{H=1,2\}}\left[J^*_y y\mu_Y(t) + \frac{1}{2}J^*_{yy}y^2\sigma_Y(t)^2 + J^*_{xy}xy\sigma_S\sigma_Y(t)\rho(t)\theta\right] \\
& + \mathbb{1}_{\{H=1\}}\kappa(t)\left[J^*(t,x,p^{1,2}(t)y,I,2) - J^*(t,x,y,I,1)\right] \\
& + \mathbb{1}_{\{H=1\}}\pi(t,H)\Big[J^*(t,x+\mathbb{1}_{\{t\le T_I\}}I,p^{1,3}(t)y,0,3) - J^*\left(t,x,y,I,1\right)\Big] \\
& + \mathbb{1}_{\{H=2\}}\pi(t,H)\Big[J^*\left(t,x+\mathbb{1}_{\{t\le T_I\}}I,p^{2,3}(t)y,0,3\right) - J^*\left(t,x,y,I,2\right)\Big]\Bigg\}
\end{aligned} \tag{A.6}$$

with terminal condition $J^*(T,x,y,I,H) = \varepsilon\frac{x^{1-\gamma}}{1-\gamma}$. Here subscripts on J denote partial derivatives. Finally, we calculate the value function J by maximizing J^* over all possible interventions at $\zeta_i = t$:

$$J(t,x,y,I,H) = \sup_{\omega_i\in\mathcal{I}_{\zeta_i}} \quad \Big\{J^*\big(t,x-\eta(\zeta_i,\omega_i,I_{\zeta_i}),y,I+\omega_i,H\big)\Big\}. \tag{A.7}$$

Note that in the case of $\omega_i = 0$ we have a continuation strategy, i.e. the family decides to keep its insurance decision. If this is optimal, then $J(t,x,y,I,H) = J^*(t,x,y,I,H)$. Consequently, there is no lump-sum payment, since we are in the no transaction region and $\eta(\zeta_i,0,I_{\zeta_i}) = 0$.

B Additional Robustness Checks

B.1 Health Shocks

[INSERT FIGURE 17 ABOUT HERE]

A health shock prevents the family from increasing the insurance protection or buying a new contract. Intuitively, one might expect that the family anticipates this restriction and buys more protection at a young age. Figure 17 compares the benchmark model with an alternative calibration without health shocks. Surprisingly, our findings document a higher insurance demand in the case without health shocks. The reason is that a health shock can be interpreted as a warning that death becomes more likely. If the wage earner faces a health shock, the family knows that he will die with a high probability in the next few years. Although an early death is clearly negative for the family, the health shock partially resolves uncertainty about the timing of dying, which itself is beneficial. With this new information the family is better able to plan consumption and investment decisions. Therefore, the family reduces consumption in order to accumulate more wealth, which can be seen in Figures 9 and 10. Due to the additional savings, the insurance demand goes down. Notice that in our benchmark calibration labor income is reduced after a health shock, which triggers a decrease in consumption. This is however also true in a calibration where the labor income is not reduced in the critical illness state. Furthermore, one might argue that the increased insurance demand without health shock results from the fact that in this setup there is no state in which the insurance acquisition is forbidden. However, the results also hold when we only consider families that do not face a health shock.

B.2 Labor Income Volatility

[INSERT FIGURE 18 ABOUT HERE]

A term insurance allows a family to (partially) hedge the risk resulting from an early death of the wage owner. Since the death of the wage earner predominately leads to a loss of labor income, the optimal insurance choice crucially depends on the labor income process. A negative feature of a long-term insurance contract is the stickiness of its premia. Consequently, such an insurance contract amplifies the effect of negative labor income shock. For instance, if a family is optimally insured and a negative labor income shock occurs, then the family has too much insurance protection given the actual income situation and must cut down on consumption. Alternatively, the family can reduce or terminate the insurance contract yielding to a loss, since term life insurance has no surrender value. In both cases, the effect of a negative labor income shock is stronger if the family has a higher insurance exposure. Therefore, hedging mortality risk comes at the cost of amplifying the effect of a negative labor income shock. In line with these findings, Figure 18 shows that the insurance demand is significantly higher for families with lower income volatility. Besides, these families also buy insurance earlier so that young families are insured as well.

B.3 Bequest Motive

[INSERT FIGURE 19 ABOUT HERE]

There is empirical evidence that bequest can be interpreted as a luxury good. In this case, households are less risk averse concerning bequest than intermediate consumption. Following De Nardi, French, and Jones (2010) and Lockwood (2012), we now study a version of our model with a more general bequest motive given by

$$u(X_T) = \varepsilon \frac{1}{1-\gamma}(k + X_T)^{1-\gamma}. \tag{B.8}$$

Notice that k and γ jointly determine the curvature of the bequest function. For $k = 0$ we obtain standard CRRA utility from bequest, which implies that households are equally risk averse concerning bequest and consumption. For $k > 0$, the marginal utility of consumption declines faster than the marginal utility of bequest implying a higher risk aversion for bequest. We recalibrate the preference parameters according to De Nardi, French, and Jones (2010). We choose $k = 273\,000$ and a bequest weight of $\varepsilon = 2.36$. Figure 19 shows that the choice of the bequest motive has a small impact on the insurance decision and the portfolio holdings. However, there is a significant reduction in consumption leading to a consumption hump around the retirement date.

B.4 Insurance Structure

[INSERT FIGURE 20 ABOUT HERE]

The structure of a term life insurance contract varies among insurance companies. Especially the fees ψ_{tr}, ψ_{ad}, the date T_C until the insurance decision can be revised and the expiration date T_I can be different. Figure 20 depicts the effect of a change of the fees. We compare the average fees of the benchmark result (grey line) to a regime with high fees (light line) and without fees (dark line). Clearly, the insurance demand decreases if fees are raised. However, although the fees are increased significantly, the decrease in insurance demand is not dramatic. This indicates that the insurance profit, captured by the fees, has a rather small effect on the insurance demand.

B.5 Family Size

[INSERT FIGURE 21 ABOUT HERE]

Figure 21 confirms the intuition that a larger family buys more insurance protection. In our model, this is captured by the consumption scaling parameter ϕ_H. The relative difference in the consumption scaling parameter in state $H = 1, 2$ and $H = 3$ is smaller, the larger the family. Consequently, for a large family more consumption is needed to obtain the same single-person equivalent utility level. If the wage earner dies, the whole income is lost, but the bigger part of consumption remains if the family size is large. This increases the insurance demand.

B.6 Risk Aversion

[INSERT FIGURE 22 ABOUT HERE]

Figure 22 shows the impact of the relative risk aversion on the average insurance sum. Apparently, risk aversion has little impact before the age of 30 and after retirement. In between, more risk averse agents demand less insurance. Hence, a more risk averse agent perceives the insurance contract as more risky compared to financial investments (stocks, bonds). However, overall the degree of relative risk aversion has only little impact on the insurance decision.

References

De Nardi, Mariacristina, Eric French, and John Jones, 2010, Why Do the Elderly Save? The Role of Medical Expenses, *Journal of Political Economy* 118, 39–75.

Lockwood, Lee M., 2012, Bequest motives and the annuity puzzle, *Review of Economic Dynamics* 15, 226–243.

	Financial Market	
μ_S	Stock drift	0.06
σ_S	Stock volatility	0.2
r	Bond drift	0.02
	Preferences	
δ	Time preference rate	0.03
γ	Relative risk aversion	4
ε	Weight of the bequest motive	1
α_{Adult}	Number of adults in the household	2
α_{Child}	Number of children in the household	1
T	Time horizon of the family	80
X_0	Initial financial wealth	38 214
	Mortality Risk	
x	Age of the wage earner at $t = 0$	20
m	X-axis displacement	89.45
b	Steepness parameter	6.5
k_1	Constant impact of a health shock	0.048
k_2	Age-dependent impact of a health shock	0.0008
	Health Shock Risk	
a	Scaling parameter	0.02489
b	X-axis displacement	66.96
c	Steepness parameter	29.42
	Income	
ξ_0	Age and education independent wage increase	0.02
b	Education dependent wage increase	0.1682
c	Education and age dependent wage increase parameter	−0.00323
d	Education and age dependent wage increase parameter	0.00002
P	Replacement ratio	0.68212
T_R	Retirement time	45
Y_0	Initial income	19 107
σ_Y^w	Volatility while working	0.2
σ_Y^r	Volatility during retirement	0
ρ	Correlation with the stock	0
$p^{1,2}(t < T_R)$	Income level after a health shock while working	0.8
$p^{1,2}(t \geq T_R)$	Income level after a health shock during retirement	1
$p^{1,3}$	Income level at death without previous health shock	0
$p^{2,3}$	Income level at death with previous health shock	0
	Insurance	
ψ_{ad}	Administrative fee	0.0299
ψ_{tr}	Transaction fee	0.0505
T_C	Latest time for changing the insurance contract	50
T_I	Contract maturity	55

Table 1: **Benchmark Calibration Parameters.**

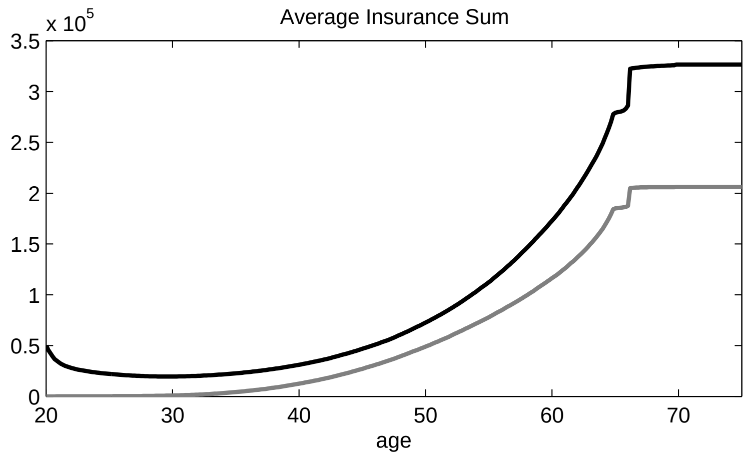

Figure 17: **Insurance Demand Compared to a Model without Health Shocks.** The dark line depicts the average insurance sum for a different biometric risk calibration without health shocks. Thus, the wage earner can only be in two health states ($A = 1, 3$) and the family can contract and increase term life insurance contracts as long as the wage earner is alive and $t < T_C$. The health shock rate and magnitude are set to zero ($\kappa = 0$, $k_1 = 0$, $k_2 = 0$) and the mortality model has a standard Gompertz structure. The parameter calibration is changed to $b = 8.9$, $m = 85.47$ to get a similar death shock distribution as in the benchmark model with critical illness shocks. The grey line represents the benchmark results including health shocks and with the original mortality parameters. The remaining parameters can be found in Table 1. The results are based on 100 000 simulations for each model.

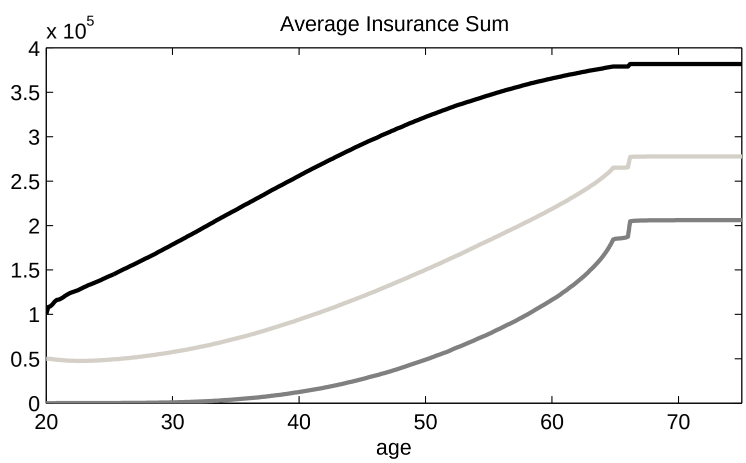

Figure 18: **Insurance Demand for Different Levels of Labor Income Volatility.** The dark line depicts the average insurance sum in a model where the labor income volatility before retirement is reduced to $\sigma_Y^w = 0.1$ and the light line depicts a reduction to $\sigma_Y^w = 0.15$. The grey line represents the benchmark results with a labor income volatility of $\sigma_Y^w = 0.2$. The remaining parameters can be found in Table 1. The results are based on 100 000 simulations for each model.

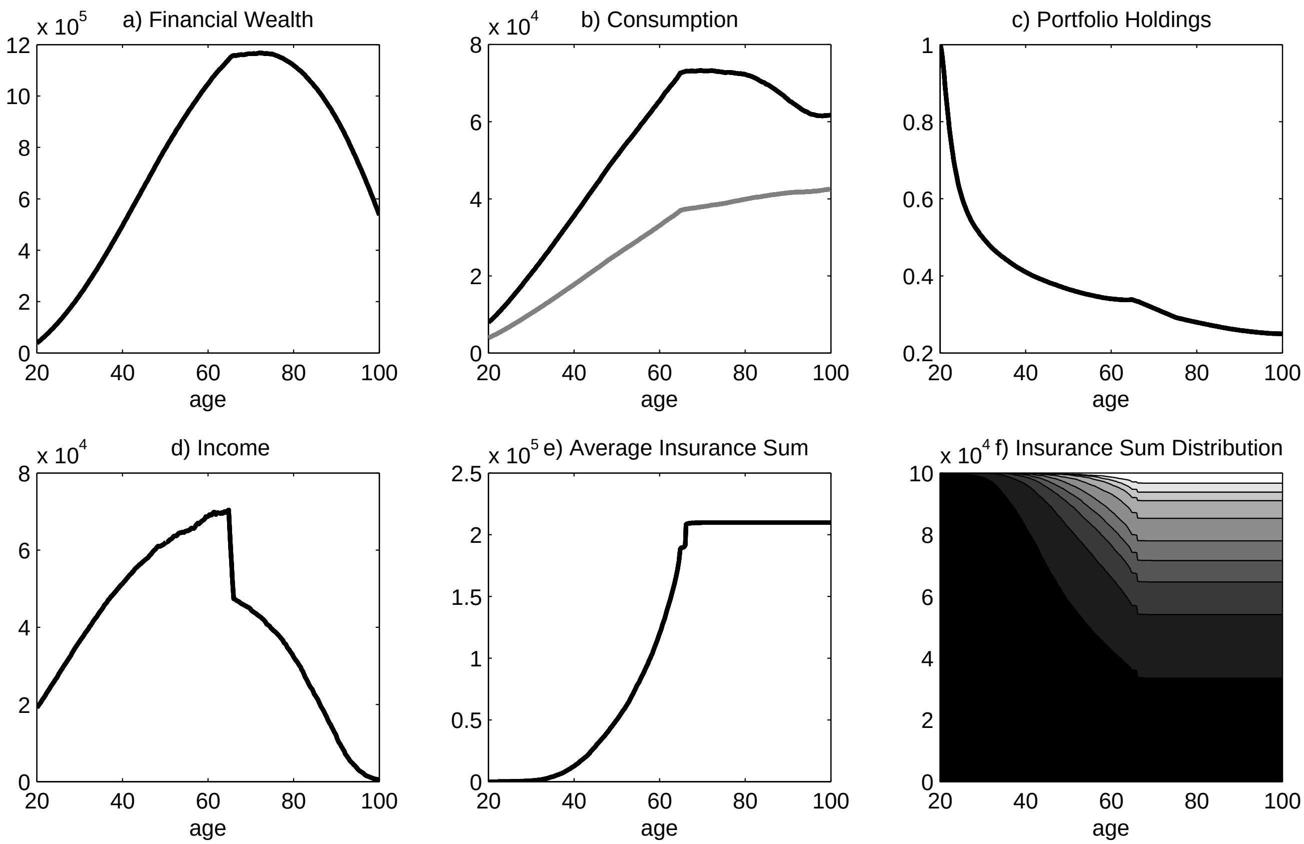

Figure 19: **Average Key Variables over the Life Cycle with Bequest as Luxury Good.** This figure depicts the average optimal control variables as well as the average financial wealth and income evolution over the life cycle based on 100 000 simulations with luxury bequest motive. The remaining parameters can be found in Table 1. a) shows the average financial wealth evolution over the life cycle. b) depicts the average optimal consumption over the life cycle. The dark line corresponds to the consumption of the family, whereas the grey line represents the equivalent level of consumption for a one person household and is scaled with the consumption scaling parameter c/ϕ. c) shows the optimal average portfolio holdings over the life cycle. d) depicts the average income of the family. e) shows the average insurance sum. f) depicts the distribution of the insurance sum over the life cycle. The darkest area marks families with no insurance contract, the white area families with the highest insurance sum (2 000 000) and the grey areas families with intermediate insurance sums.

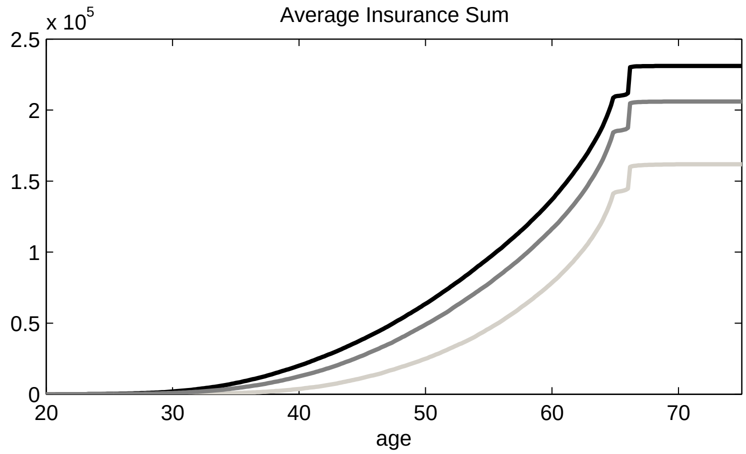

Figure 20: **Insurance Demand for Different Fees.** The light line depicts the average insurance sum in a model where the fees of the insurance company are increased such that the administrative fee is $\psi_{ad} = 12.68\%$ and the transaction fee equals $\psi_{tr} = 13.62\%$. The values are taken from "map-report no. 807-808" and represent the highest fees in the German life insurance market. The dark line is for an actuarially fair insurance without fees ($\psi_{ad} = 0$, $\psi_{tr} = 0$). The grey line represents the benchmark results with fees of $\psi_{ad} = 2.99\%$ and $\psi_{tr} = 5.05\%$ that correspond to average fees in the German life insurance market. The remaining parameters can be found in Table 1. The results are based on 100 000 simulations for each model.

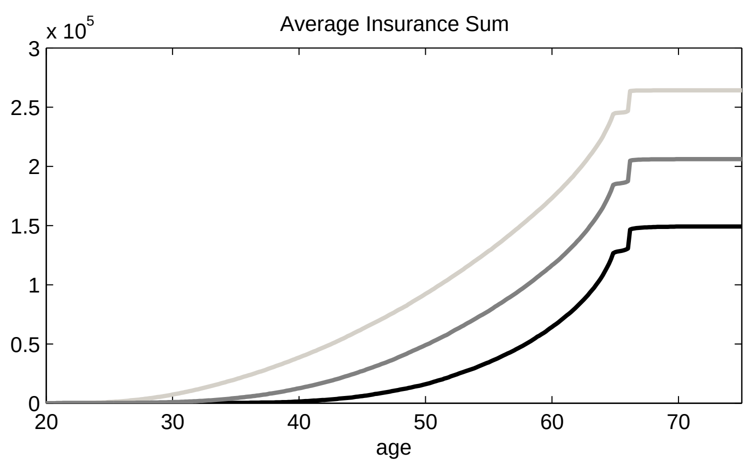

Figure 21: **Insurance Demand for Different Family Sizes.** The dark line depicts the average insurance sum over the life cycle for a family without children. The consumption scaling parameter are changed to $\phi_{1,2} = 1.6245$ and $\phi_3 = 1$. The light line gives the insurance demand with three children and the corresponding parameters are $\phi_{1,2} = 2.6850$, $\phi_3 = 2.2078$. The grey line represents the benchmark results for a family with one child and consumption scaling parameters are $\phi_{1,2} = 2.0043$ and $\phi_3 = 1.4498$. The remaining parameters can be found in Table 1. The results are based on 100 000 simulations for each model.

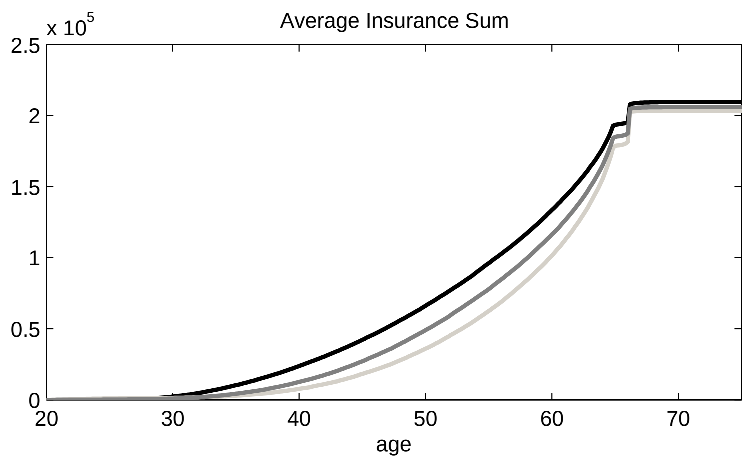

Figure 22: **Insurance Demand for Different Risk Aversions.** The figure depicts the average insurance sum over the life cycle for different values of the relative risk aversion. The dark line shows results for a low level of relative risk aversion ($\gamma = 3$), the grey line corresponds to the benchmark case with $\gamma = 4$ and the light line presents a more risk averse agent with $\gamma = 5$. The remaining parameters can be found in Table 1. The results are based on 100 000 simulations for each model.